The Comparative Study of Tempereature Measuring of The Hottest Fuel Rod In The Nuclear Power Plant of Bushehr City In Iran

Mousavi Shirazi* **and Seyed Alireza**

**M.Phil Graduated in Nuclear Reactor Engineering, Member of scientific board, Department of Physics,*

Islamic Azad University, South Tehran Branch, Tehran, Iran

Email: alireza_moosavi@yahoo.com (Tel: +98-9122027059, Fax: +98-21-33060909)

ABSTRACT

In this paper have been determined the effects of temperature distribution at the hottest fuel rod (Hot Fuel Pin) in the Bushehr nuclear power plant for defined thermal conductivities from different mathematical functions. Whereas Hot Fuel Pin is one of the most fuel rod from aspect of heat temperature distribution in core of nuclear power plant, thus in this article have been obtained the values of average temperature of fuel rod (T_{ave})and temperatures of inner and outer diameters of fuel rod (T_{fi}, T_{fo}) have been obtained truly for a cylindrical fuel. For calculation of temperature have been used two methods means: analytical and numerical methods and in the numerical method has been used from computer programming (Turbo Pascal). Moreover calculations of temperatures have been performed for two stages and results have been modified than previous stage and have been compared together. In every stage of calculations by two said methods, the values of thermal conductivity (k) have been obtained from two methods means: Lagrange and finite difference methods and their related mathematical functions.

ABSTRAK

Dalam kertas ini telah ditentukan kesan-kesan taburan suhu di rod bahan api paling hangat (Hot Fuel Pin) dalam logi kuasa nuklear Bushehr untuk conductivities terma ditakrifkan dari fungsi-fungsi matematik berbeza. Manakala Hot Fuel Pin ialah salah satu paling rod bahan api dari aspek taburan suhu haba dalam teras logi kuasa nuklear, oleh itu dalam makalah ini telah diperolehi nilai-nilai suhu purata rod bahan api (T_{ave}) dan suhu-suhu garis pusat dalam dan luar rod bahan api (T_{fi} T_{fo} ,) telah diperolehi benar-benar untuk satu bahan api berbentuk silinder. Untuk pengiraan suhu telah digunakan dua cara cara: kaedah-kaedah analisis dan berangka dan dalam kaedah berangka telah digunakan dari pengaturcaraan komputer (Turbo Pascal). Tambahan pula pengiraan-pengiraan suhu-suhu telah dipersembahkan untuk dua peringkat dan keputusan-keputusan telah diubahsuai daripada peringkat sebelumnya dan telah dibandingkan bersama. Dalam setiap peringkat pengiraan-pengiraan oleh dua kata kaedah-kaedah, nilai-nilai kekonduksian terma (k) telah diperolehi dari dua cara cara: Lagrange dan kaedah-kaedah beza terhingga dan fungsi-fungsi matematik berkaitan mereka.

Keywords: *Interpolation equations, temperature, hottest fuel rod, nuclear power plant, thermal conductivity coefficient*.

INTRODUCTION

For a nuclear reactor from VVER-1000 type which operating on steady state and stuff of its fuel rod is UO_2, is surrendered from changes of temperature in tension of axial (z) and angle. Also is supposed [1] $T_{ave} = 1800(K)$ (Final safety Analysis Report, 2003) and said rod is the hottest fuel rod in the nuclear reactor core to epithet: Hot Fuel Pin.

There are below conditions:

Radius inside of fuel rod gap = r_i = 0.75 mm (Final safety Analysis Report, 2003)

Radius outside of fuel rod = r_o =3.785 mm (Final safety Analysis Report, 2003)

Height of fuel rod = 3.53 m (Final safety Analysis Report, 2003)

Linear power for hottest fuel rod = 448 W/cm (Final safety Analysis Report, 2003)

Boundary conditions:

1) If: $r = r_i$ then: [2] $T = T_{fi}$, $\frac{\partial T(r,\varphi,z)}{\partial r} = 0$

2) If: $r = r_o$ then: [3] $T = T_{fo}$

Fig.1. Showing of fuel rod sidelong surface

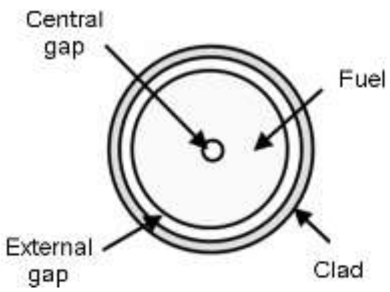

Fig.2. Showing of fuel rod upper surface

MATERIALS AND METHODS

By using of interpolation equations from two methods: Lagrange and finite difference methods, is calculated [4] k which by its using and by considering of above initial conditions, is defined function of temperature distribution.

From said function, value of T_{ave} determinates and is compared between initial supposition. In the next stage, average temperature (T_{ave}) which had obtained from temperature distribution function, puts instead of supposed average temperature in the first, means: $T_{ave} \approx \frac{T_{fi} + T_{fo}}{2} = 1800(K)$ (1)

and the said procedure so is repeated that finally acquires accurate determined value of T_{ave}. Moreover determination of T_{ave}'s accurate value, is determined accurate value of inside and outside diameters of fuel rod means: T_{fi}, T_{fo}.

Method of solution: In cylindrical coordinates, heat transfer equation is to below form:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{(Wakil, 1991)} \qquad (2)$$

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[1] Average temperature of fuel rod [2] Temperature of inner diameter of fuel rod [3] Temperature of outer diameter of fuel rod

[4] Thermal conductivity coefficient

Now one time based on analytical method and the next time from the numerical method, is solved above equation and values of T_{ave}, T_{fo}, T_{fi} and temperature distribution function are determined in the hottest fuel rod of Bushehr atomic power plant in Iran and are compared together.

Calculations of temperature distribution from analytical method

By supposition of insignificance of temperature changes in axial tension (z) and also in angular tension (φ) can be writing:

$$\frac{\partial^2 T}{\partial z^2} = \frac{\partial^2 T}{\partial \varphi^2} = \frac{\partial T}{\partial t} \cong 0 \qquad (3)$$

Thus: $$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = -\frac{\dot{q}'''}{k} \qquad (4)$$

So, equation of T is produced by using of above condition and equation:

$$T = -\frac{\dot{q}''' r^2}{4k} + C_1 \ln r + C_2 \qquad (5)$$

By using of below boundary condition, can be produced values of C_1, C_2 and then is determined equation 5 as definite equation format. According to boundary condition 1:

$$C_1 = \frac{\dot{q}''' r^2}{2k} \qquad (6)$$

$$C_2 = T_{fi} + (2.164335 \times 10^{-6}) \frac{\dot{q}'''}{k} \qquad (7)$$

For determination of numeric values of C_1 and C_2, are requirement values of $\dot{q}'''$, T_{fi} and k.

Whereas linear heat rate in Hot Fuel Pin was: $\dot{q} = 448\ (W/cm)$ (Final safety Analysis Report, 2003), heat flux for Hot Fuel Pin will be: $\dot{q}''' = 103.6\, KW/m^3$ (8)

Also method of T_{fi}'s calculation by considering of conduction heat transfer formula is:

$$-kA\frac{\partial T}{\partial r} = \dot{q}''' V \quad \text{(Neil and Kazimi, 1990)} \qquad (9)$$

where: A is sidelong area of fuel rod. According to boundary condition 2:

In $r = r_o$: $T = T_{fo}$ so: $$kA\frac{T_{fi} - T_{fo}}{r_o - r_i} = \dot{q}'''\pi(r_o^2 - r_i^2)L \qquad (10)$$

Thus by using of this supposition: $T_{ave} \approx \dfrac{T_{fi} + T_{fo}}{2}$, can be writing:

$$k(2\pi r_o L)\frac{2(T_{fi} - T_{ave})}{r_o - r_i} = \dot{q}'''\pi(r_o^2 - r_i^2)L \qquad (11)$$

Now value of T_{ave} is obtained by using of temperature distribution function and its integrity (formula 13) and is compared with T_{ave} which had been supposed in first, so:

$$T_{ave} = \frac{1}{v}\iint T(r)dv \quad \text{(Olander, 1976)} \qquad (12)$$

$$T_{ave} = \frac{1}{\pi(r_o^2 - r_i^2)L}\int_{z_1=-\frac{L}{2}}^{Z_2=\frac{L}{2}}\int_{r_i}^{r_o} T(r)2\pi r dr dz \qquad (13)$$

Calculation of k from Lagrange method

Firstly is considered produced value from Lagrange method. The interpolation to Lagrange method is calculated by information of table 1and from this method:

$$P_n(x) = \sum_{m=0}^{n} f(x_m) L_{n,m}(x) \qquad m = 0,1,\ldots,n \text{ (Dusinberre, 1961)} \tag{14}$$

$$L_{n,m}(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{m-1})\cdots(x-x_{m+1})\cdots(x-x_n)}{(x_m-x_0)(x_m-x_1)\cdots(x_m-x_{m-1})\cdots(x_m-x_n)} \tag{15}$$

Table 1: The changes of thermal conductivity coefficient of fuel with temperature for the fuel rod in Bushehr atomic power plant in Iran (Brancharia, 1966; Final safety Analysis Report, 2003)

Temperature (K)	Thermal Conductivity Coefficient (k) in (W/m.K)
300	8.15
1100	3.75
1700	2.50
2700	2.65
3100	3.50

Now in this part by having of T_{ave_1}, this value is put instead of conjectural value of T_{ave} and by considering of T_{ave_1}, values of thermal conductivity coefficient (k_2), T_{ave_2}, T_{fi_2} and T_{fo_2} are calculated again according to done methods in the last stage.

Calculation of k from finite difference method

In this stage, value of k is calculated from finite difference method and by using of related information to table 1 and consider to supposed T_{ave} in first. The interpolation to finite difference method is calculated through this formula:

$$P_{a,b,c,\ldots,j}(x) = f_a + f_{a,b}(x-x_a) + f_{a,b,c}(x-x_a)(x-x_b) + \cdots + f_{a,b,c,\ldots,j}(x-x_a)(x-x_b)(x-x_c)\cdots(x-x_{j-1})$$

$$f_{a,b,c,\ldots,j} = \frac{f_{b,c,\ldots,j} - f_{a,b,c,\ldots,(j-1)}}{x_j - x_a} \qquad \text{(Dusinberre, 1961)} \tag{16}$$

Now by having T'_{ave_1}, are calculated again value of k'_2 and values of T'_{ave_2}, T'_{fi_2} and T'_{fo_2} too, according to performed methods in the last stage.

Calculation of temperature distribution from numerical method

In this stage by using of numerical method and solving the equation 2 and comparing its results with analytical method from numerical method, the values of

$$\frac{\partial T}{\partial r}, \frac{\partial^2 T}{\partial r^2} \text{ are: } \frac{\partial T}{\partial r} = \frac{T_{n+1} - T_{n-1}}{2\Delta r} \qquad \text{(Mitchell and Griffiths, 1980; Smith, 1978)} \tag{17}$$

and

$$\frac{\partial^2 T}{\partial r^2} = \frac{T_{n+1} - 2T_n + T_{n-1}}{(\Delta r)^2} \qquad \text{(Mitchell and Griffiths, 1980; Smith, 1978)} \tag{18}$$

Thus equation 2 changes to this format:

$$\left(\frac{T_{n+1} - 2T_n + T_{n-1}}{(\Delta r)^2}\right) + \frac{1}{r}\left(\frac{T_{n+1} - T_{n-1}}{2(\Delta r)}\right) + \frac{\dot{q}'''}{k} = 0 \tag{19}$$

and also:

$$\Delta r = r_{n+1} - r_n \tag{20}$$

If: $\Delta r = 10^{-5} m$, between distance of inside and outside diameters of fuel rod is divided to small intervals means: Δr and by a computer program (Turbo Pascal) are produced values of temperature in points: $r = r_i = 75\Delta r$ and $r = r_o = 3785\Delta r$ (Shoichiro, 1991) means: T_{fi}, T_{fo}. Therefore by programming are determined values of T_{fi}, T_{fo} and T_{ave} for numerical method the same of analytical method for two stages.

RESULTS AND DISCUSSION

The comparison all of the produced parameters for Hot Fuel Pin, are produced in the following tables (Tables 2, 3, 4 and 5):

Table 2: Evaluation of results in the first stage by numerical and analytical methods

Percent of error	The first stage of calculations from numerical method (Defined *k* from finite difference method) $k_1' = 2.475(W/mK)$	The first stage of calculations from analytical method (Defined *k* from Lagrange method) $k_1 = 2.564(W/mK)$
3.05%	T_{ave_1} =1822.5(K)	T_{ave_1} =1880.0(K)
-1.23%	T_{fi_1} =2048.6(K)	T_{fi_1} =2023.7(K)
1.61%	T_{fo_1} =1708.3(K)	T_{fo_1} =1736.3(K)
		$T_1 = -10101.4r^2 + 20202.8r^2 \ln r + 1952$

Table 3: Evaluation of results in the first stage by numerical and analytical methods

Percent of error	The first stage of calculations from numerical method (Defined *k* from Lagrange method) $k_1 = 2.564(W/mK)$	The first stage of calculations from analytical method (Defined *k* from finite difference method) $k_1' = 2.475(W/mK)$
5.75%	T'_{ave_1} =1802.2(K)	T'_{ave_1} =1912.2(K)
3.54%	T'_{fi_1} =2023.7(K)	T'_{fi_1} =2098.0(K)
1.47%	T'_{fo_1} =1701.0(K)	T'_{fo_1} =1726.4(K)
-	-	$T'_1 = -10464.6r^2 + 21082.6r^2 \ln r + 2098$

Table 4: Evaluation of results in the second stage by numerical and analytical methods

Percent of error	The second stage of calculations from numerical method (Defined *k* from finite difference method) $k_2' = 2.489 (W/mK)$	The second stage of calculations from analytical method (Defined *k* from Lagrange method) $k_2 = 2.475 (W/mK)$
5.48%	T_{ave_2} =1807.4(K)	T_{ave_2} =1912.2(K)
2.95%	T_{fi_2} =2036.0(K)	T_{fi_2} =2098.0(K)
1.35%	T_{fo_2} =1703.0(K)	T_{fo_2} =1726.4(K)
		$T_2 = -10464.6r^2 + 21082.6r^2 \ln r + 2098$

Table 5: Evaluation of results in the second stage by numerical and analytical methods

Percent of error	The second stage of calculations from numerical method (Defined *k* from Lagrange method) $k_2 = 2.475 (W/mK)$	The second stage of calculations from analytical method (Defined *k* from finite difference method) $k_2' = 2.489 (W/mK)$
4.01%	T'_{ave_2} =1822.5(K)	T'_{ave_2} =1898.8(K)
-0.59%	T'_{fi_2} =2048.6(K)	T'_{fi_2} =2036.4(K)
3.00%	T'_{fo_2} =1708.3(K)	T'_{fo_2} =1761.2(K)
		$T'_2 = -10405.8r^2 + 20811r^2 \ln r + 2036.4$

These tables show results from two methods: Lagrange and finite difference. Values of T_{ave}, T_{fi} and T_{fo} in every time computation are convergent to the last stage and are modified in every stage than previous stage. One of the most important results is the defined value of *k* in the second stage of Lagrange method is equal with the defined value of *k* in the first stage of finite difference method means: $k_1' = k_2$.

The next result is that obtained temperatures in the first stage from computation of analytical method and defined *k* from Lagrange method are more than obtained temperatures from numerical method and this matter is seen in the second stage too, but in the second stage the values of temperatures are more convergent than the first stage and hereby is determined the values of temperatures (T_{ave}, T_{fi} and T_{fo}) truly in the Hot Fuel Pin.

The results show while be considered the defined *k* from finite difference method then in the first stage of calculation the obtained values of temperatures from analytical and numerical methods are convergent together (unlike of use of defined *k* from Lagrange method), but in the second stage of calculation are diverged the values of temperature than the first stage. Therefore can be saying the suitable determined values for T_{ave}, T_{fi} and T_{fo} of Hot Fuel Pin are values of temperatures in the second stage by defined *k* from Lagrange method and in the first stage by defined *k* from finite difference method.

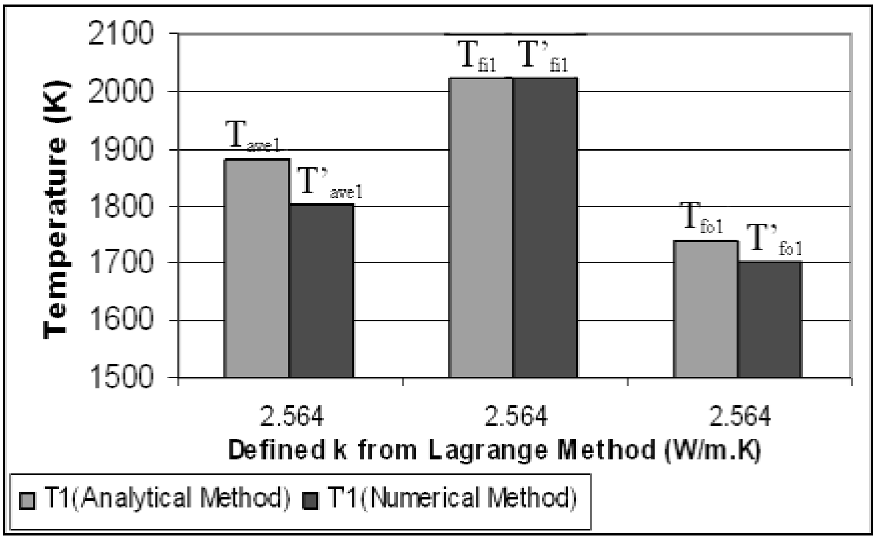

Fig.3. The obtained temperatures from analytical and numerical methods in the first stage by defined *k* from Lagrange method

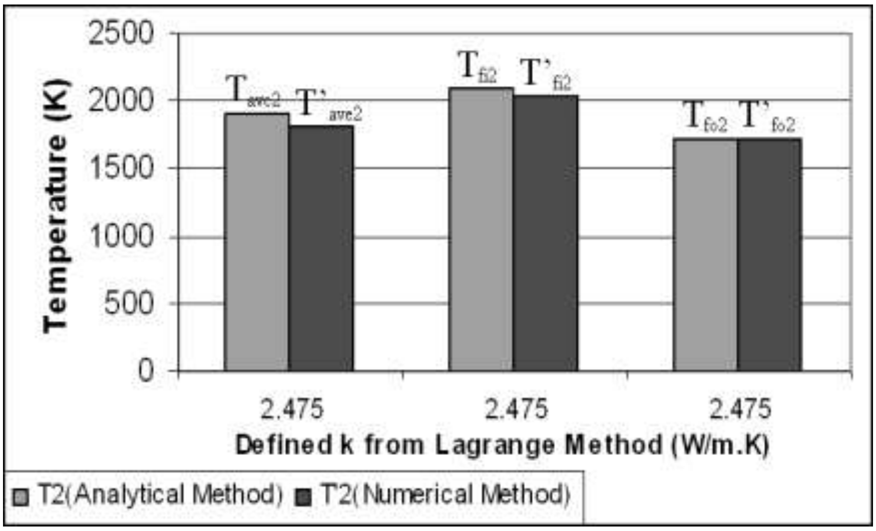

Fig.4. The obtained temperatures from analytical and numerical methods in the second stage by defined *k* from Lagrange method

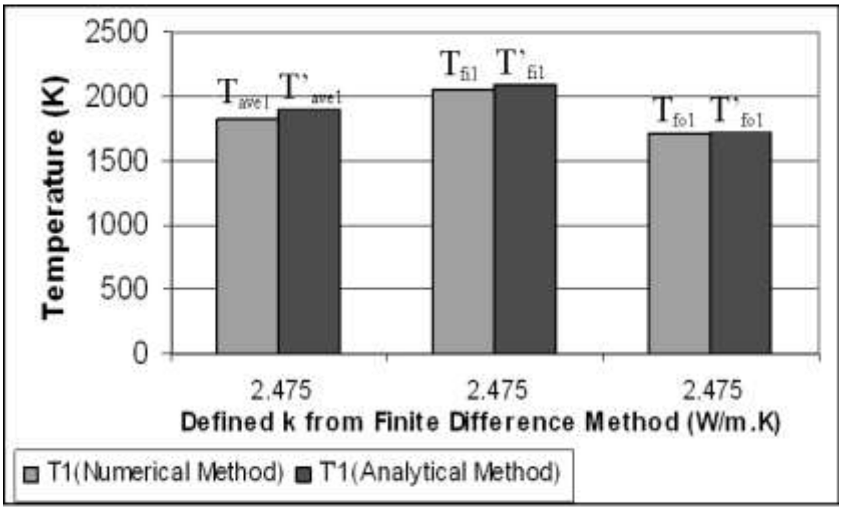

Fig.5. The obtained temperatures from numerical and analytical methods in the first stage by defined *k* from finite difference method

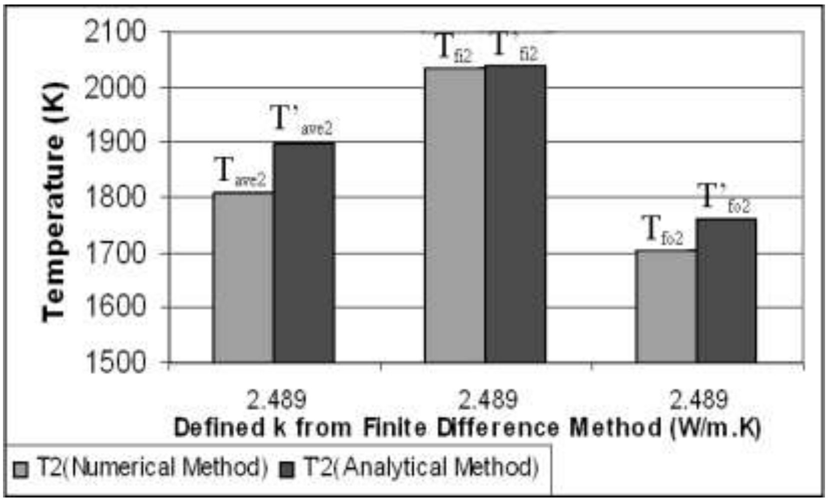

Fig.6. The obtained temperatures from numerical and analytical methods in the second stage by defined *k* from finite difference method

CONCLUSION

This work has shown the effects of temperature distribution at the hottest fuel rod (Hot Fuel Pin) in the Bushehr nuclear power plant for defined thermal conductivities from different mathematical functions. For calculation of temperature have been used two methods means: analytical and numerical methods and in the numerical method has been used from computer programming (Turbo Pascal). Moreover calculations of temperatures have been performed for two stages and results have been modified than previous stage and have been compared together. In every stage of calculations by two said methods, the values of thermal conductivity (k) have been obtained from two methods means: Lagrange and finite difference methods and their related mathematical functions.

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