Microscopic Preformation Factor of Alpha-decay Process in Compound Nuclei; A New Perspective

Saad M. Saleh, Redzuwan Yahaya and Shahidan Radiman

School of Applied Physics, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 Bangi. Selangor. Malaysia

Email: saadtm2000@gmail.com

ABSTRACT

The preformation factor of alpha-decay process in compound nuclei is microscopically proposed with a new perspective. The formation of alpha particle inside the parent nuclei is considered as a quantum-mechanical state which is yielded from a certain interaction among the valance nucleons. This interaction is very similar to that one responsible for the formation of the quasi-bound states in many-body system. This introduced microscopic perspective might give more insight to the understanding of the nuclear structure in the compound nuclei.

Keywords: *alpha particle, preformation factor, nuclear structure*

INTRODUCTION

ne of the most important process that has been used since a long time to study the nuclear structure of heavy nuclei is alpha decay. This is because of many reasons related to its domination, experimental measurements, and the available microscopic theory that interprets it. This process spontaneously occurs in most heavy and super heavy nuclei and it is one of the most dominant modes in them. The simplicity of this process that is due to the small number of observables, the decay width and the energy of emitted alpha particle, and some properties of its transition, from ground state to another ground state with same parity. As well as, the experimental measurements are available with high resolution. These reasons have pushed researches to microscopically understand shell closure, and stability of nuclei that formed what is called islands of stability, around Z ~ 114 and N ~ 184, by different theories and models. (Sushil Kumar et al. 2009a, Jianmin et al. 2010, Santhosh et al.(2009), Dongdong and Zhongzhou (2009), Zhang et al. (2009), Chowdhury et al.(2006) ,Seif (2006), Jianmin et al. 2010, DONG Jian-Min et al. (2009), Mohammad Shoeb and Sonika (2009), and SILI et al. (2005)).

Theoretically, Gamow and Condon and Gurney (1928) were the first who reported the explanation of α-decay as a quantum tunneling effect. Usually, the α-decay process is described as a preformed α-particle tunneling through a potential barrier between the cluster and the daughter nucleus. The absolute α-decay width is mainly determined by the preformation factor of the alpha cluster inside the parent nucleus and the α-cluster penetration probability.the penetrability or the penetration probability can be determined by the wellknown Wentzel–Kramers–Brillouin (WKB) semiclassical approximation. However, the prefomation factor (cluster formation probability and also called the spectroscopic factor) is defined as the quantum-mechanical probability of finding the cluster inside the parent nucleus (Dongdong and Zhongzhou (2009), Chang Xu and Zhongzhou Ren (2005), Sham et al. (1997)).

Since the theoretical approach of Gamow and Condon and Gurney was presented, the widths or half-lives of alpha-decay, with the use of this approach, have being reproduced for unstable nuclei (compound nuclei) by using different microscopic models to provide more understanding to the nuclear structure. Early calculations adopted the Gamow theory with missing the effect of the cluster formation by assuming a value of 1 for the preformation factor (Buck et al. (1992, 1993, 1994) , Xu and Pei (2006), Routray el al. (2009)). Then, many calculations assumed the preformation factor is a quantum-mechanical probability of a value different to each nucleus and less than one. This value could easily be found by dividing the experimental value of the alpha-decay rate by the penetration factor of the nucleus. This assumption improved the reproduction of alpha-decay widths (Lovas et al. (1998)). But, it did not add any more sensitive understanding to the microscopic theory of the models used in this process because the potentials of the nuclear structure had contributions only in the penetration factors although the resonance state and the assault frequency were considered. As well as, the parameters of the suggested potential were mostly determined by fitting to the experimental values of alpha-particle energy. Therefore each different potential can produce different preformation factors. So, the preformation factor has become a problem because it has not been estimated by a microscopic theory (Chang Xu and Zhongzhou Ren (2005),

Madhubrata Bhattacharya and Gangopadhyay (2008), Dongdong and Zhongzhou (2009), Jianmin et al. 2010, Mohammad Shoeb and Sonika (2009), Zhang et al. (2009)).

In our work we present a new perspective to microscopically describe the preformation of alpha clusters inside the compound parent nuclei. This formation effect is proposed as a quantum-mechanical state and the preformation factor is found by the use of Schrodinger equation and the use of the formation energy which derived from the binding energy.

PARENT-NUCLEUS PREFORMATION STATES

The total wave function of a system as a nucleus can be written as a sum of all quantum-mechanical states (excited or non-excited) due to all types of interactions responsible for the formation of these states. So, these interactions could be written as a sum of different terms. Each term could be responsible for formation a certain type of quantum-mechanical states to describe an effect and to calculate, directly or indirectly, the observables of this effect. Actually this assumption is not new, because it is already done in quantum mechanics when the wave function of single particle states are found by using the harmonic oscillator potential energy in the Hamiltonian operator and solving Schrodinger equation. This potential does not represent all interactions among the particles inside the system and the harmonic oscillator wave functions do not describe all particles in the system. This is why the residual interactions are added to the oscillator potential in the Hamiltonian operator to describe any other effects in many-body system by a correction to the eigenvalues of energy and building new wavefunctions within the perturbation theory.

For alpha-decay process, one can describe this effect in two steps of interactions. In the first step, the alpha cluster is preformed inside the compound nucleus by a certain interaction which is in charge of only this preformation. In the second, the preformed alpha cluster is emitted from the parent nucleus by another interaction. This emission process is well described within two-body system in literature review. In compound nuclei, there are many possibilities of formations of different clusters, but for simplicity, we assume for this effect that there are only two possible states; one is for the parent-nucleus formation state ϕ_o and the other for the alpha-particle formation state inside the nuclear matter (parent nucleus) ϕ_1. The wave function of all possibilities of any cluster formation ψ_p can be written as

$$\psi_p = a_o \phi_o + a_1 \phi_1 \tag{1}$$

Where a_o is the amplitude of the of the parent formation state without any other formation of any cluster and a_1 is the amplitude of the alpha-cluster formation state. The idea of formation the cluster inside the parent nucleus was potentially studied and emphasized in literature review [wildermuth and Tang (1977)]. And the clustering effect played a significant role in the improvement of the calculations of alpha-decay widths (Lovas et al. (1998), Chang Xu and Zhongzhou Ren (2005), Madhubrata Bhattacharya and Gangopadhyay (2008), Dongdong and Zhongzhou (2009)).

The Hamiltonian operator H_f for these states consists of the potential responsible for the formation of the clusters. In many-body system, it is so difficult to get a form for this potential and any other nuclear potential from the available nuclear model (Chang Xu and Zhongzhou Ren (2005), Dongdong and Zhongzhou (2009)). In the preformed cluster model the cluster states are derived from the fragmentation theory in which the cluster states are functions of dynamical collective coordinates of mass and charge asymmetries (Sham et al. (1997), Sushil Kumar et al. (2009)).

If one sets this Hamiltonian in Schrodinger equation, it is important to think about the eigenvalues of this Hamiltonian. One can assume that there is an amount of energy required to form a cluster anywhere. This energy can be called the formation energy E_f which can be defined as the work needed to bring the individual nucleons together and form a cluster. So, Schrodinger Equation can be written as

$$H_f \psi_p = E_f \psi_p \tag{2}$$

Therefore there will be a formation energy for each type of cluster and the total formation energy of the parent nucleus is equivalent to the sum of all formation energies of the possible clusters. For our assumption in Eqs.(1and 2), there are two formation energies; E_{fo} for the state ϕ_o, and $E_{f\alpha}$ for the state ϕ_1. So, one can write

$$E_f = E_{fo} - E_{f\alpha}, \tag{3}$$

$$H_f \phi_o = E_{fo} \phi_o, \tag{4}$$

and,

$$H_f \phi_1 = E_{f\alpha} \phi_1, \tag{5}$$

where $E_{f\alpha}$ is with opposite sign to other formation energies. This will be explained in the next section.

In accordance to Eq.(1) and the orthonormalization condition, one can define that $|a_0|^2$ is the probability density of finding the parent nucleus without any cluster formed inside it and $|a_1|^2$ the probability density of finding the alpha particle inside the nucleus. So, the total probability density of the nucleus $|\psi_p|^2$ is one, and

$$1 = |a_o|^2 + |a_1|^2 \tag{6}$$

If we suppose that a_o and a_1 are time independent, it will be simply to assume that Eq.(1) is as an equation of formation distribution between two states. It can be defined as that the total formation probability (which is one) of finding a compound nucleus in its ground state is the sum of two probabilities in the two states mentioned above. So, the preformation factor of alpha cluster, P_o can be as the probability of the alpha-particle formation as follows

$$P_o = |a_1|^2 \tag{7}$$

FORMATION ENERGY AND FORMATION-STATE ENERGY

As mentioned above that the formation energy is the energy required to form a cluster of any system from its constituents. According to this definition, the binding energy of a nucleus is the same as the formation energy. So, one can easily obtain the formation energy of alpha particle from the calculations of the binding energy which is mainly the mass defect between the alpha particle and the free four nucleons. The formation energy of alpha particle inside the nuclear matter is absolutely different and less than that outside the nuclear matter. This is because the nucleons inside the nuclear matter are already close to each other. The energy of quantum-mechanical formation state $E_{f\alpha}$ is the formation energy inside the nuclear matter. This formation state energy can be calculate as a difference in two binding energies, B_{last4n} and $B_{\alpha-4n}$, as

$$E_{f\alpha} = B_{last4n} - B_{\alpha-4n} \tag{8}$$

where B_{last4n} is calculated by the sum of the separation energies of first last nucleon, second last, third last and forth last, as,

$$B_{last4n} = S_{A^{th}n} + S_{(A-1)^{th}n} + S_{(A-2)^{th}n} + S_{(A-3)^{th}n} \tag{9}$$

where $S_{A^{th}n}$ is mainly given by the mass of a nucleus of a mass number A and the nucleon mass M_n as

$$S_{(A)^{th}n} = M(A-1) + M_n - M(A) \tag{10}$$

The second binding energy in Eq.(8) is calculated by

$$B_{\alpha-4n} = S_\alpha + B_\alpha, \tag{11}$$

where S_α is the separation energy of alpha particle from the parent nucleus, and B_α is the binding energy of alpha particles.

The two terms of binding energies in Eq.(8) seems same because in both we calculate the binding energy of same initial system, the parent nucleus, and same final system, the daughter and the four free nucleons. But they are different because in the first one, Eq.(9), the formation of alpha particle is not included whereas in the second it is.

Finally, the energy of alpha-particle formation state inside the nuclear matter can be found from Eq.(8). This equation is written in this way to insure that the formation energy is positive, in addition, we here like to refer that all values of binding energies in all equations above are assumed to be opposite sign.

PREFORMATION FACTOR

Now, the preformation factor can simply be derived from the Eq.(1) without any need to form the Hamiltonian operator or solving Schrodinger equation. This will be simply done by using the energy eigenvalues which can be found from previous section by Eq.(8). If we substitute Eq.(1) in Eq.(2) and multiply it from left by ψ_p^* then integrate over the whole space that describes the eigenstates, we obtain, in Dirac notation,

$$\left\langle \psi_p \middle| H_f \middle| \psi_p \right\rangle = E_f = |a_o|^2 \left\langle \phi_o \middle| H_f \middle| \phi_o \right\rangle + |a_1|^2 \left\langle \phi_1 \middle| H_f \middle| \phi_1 \right\rangle \tag{12}$$

when we use Eqs.(4, 5 and 6) and substitute for E_{fo} from Eq.(3), we obtain

$$|a_1|^2 = P_o = \frac{E_{f\alpha}}{E_f} \tag{13}$$

RESULTS, DISCUSSION AND CONCLUSION

One of the best typical nuclei that can be always used to evaluate any new model or method of calculation is Po^{212} because this nucleus is made of two protons and two neutrons after the closed shell. When we calculated the formation energy of Po^{212} nucleus by using experimental date from Audia et. Al (2003), we found that the formation energy of alpha particle inside the nuclear is about 17.18972MeV and the preformation factor P_o is 0.0103. Varga et al. (1992) presented the value of $P_o = 0.3$ suitable for the combination between the cluster model and the shell model. This value was greater than the previous one, 0.01, obtained from the shell model calculation by Blendowske et al. in 1991. The value of 0.3 was also obtained after potential work was done to combine BCS method with shell model and cluster model (Lovas et al. (1998)). Zhang et al. (2009) obtained 0.38 for even–even heavy nuclei.

Although the above comparison shows a reduction in our calculated preformation factor, but this smaller value might be suitable with some other nuclei and with some considerations that could be taken in the calculation of the formation energy from that part of last nucleon energy separation in Eq.(9). This equation can be calculated in different ways depending on the assumption of which nucleon (a proton or a neutron) is separated first.

REFERENCES

Audia G., A.H. Wapstrab and C. Thibaulta (2003), The AME2003 atomic mass evaluation (II). Tables, graphs and references, Nuclear Physics A 729 (2003) 337–676.

Buck B., Merchant A.C., Perez S.M., (1992), Favoured alpha decays of odd-mass nuclei, J. Phys. G 18;143.

Buck B., Merchant A.C., Perez S.M., (1992), decay calculations with realistic potential, Phys. Rev.Lett. 45 ; 2247.

Buck B., Merchant A.C., Perez S.M.,(1993), Half-lives of alpha favored decays from nuclear ground states, At. Data Nucl. Data Tables 54; 53.

Buck B., Merchant A.C., Perez S.M.,(1994), Alpha-cluster structure in 212Po, Phys. Rev. Lett. 72 ; 1326.

Chang Xu , Zhongzhou Ren (2005), Favored α-decays of medium mass nuclei in density-dependent cluster model, Nuclear Physics A 760; 303–316.

Chang Xu a,∗, Zhongzhou Ren a(2005), Favored α-decays of medium mass nuclei in density-dependent cluster model, Nuclear Physics A 760 (2005) 303–316.

Chowdhury P.R., Samanta C., Basu D.N.(2006), α-decay half-lives of new superheavy nuclei, Phys. Rev. C 73; 014612 .

DONG Jian-Min, ZHANG Hong-Fei, ZUO Wei, WANG Yan-Zhao, and LI Jun-Qing (2009), Gound State Properties of Nuclei in 295118 α-Decay Chain, Commun. Theor. Phys. (Beijing, China) 52; pp. 1055–1061

Dongdong Ni, Zhongzhou Ren,(2009), α-Decay calculations of medium mass nuclei within generalized density-dependent cluster model, Nucl. Phys. A 828 348–359.

Dongdong Ni, Zhongzhong Ren (2009), Microscopic Calculation of alpha-decay half-lives within the cluster model, Nucl. Phys. A825;145-158.

Jianmin Dong, Hongfei Zhang , Yanzhao Wang ,Wei Zuo a,b, Junqing Li a(2010), Alpha-decay for heavy nuclei in the ground and isomeric states, Nuclear Physics A 832 (2010) 198–20.

Jianmin Dong x, Hongfei Zhang , Yanzhao Wang , Wei Zuo , Junqing Li (2010), Alpha-decay for heavy nuclei in the ground and isomeric states , Nuclear Physics A 832; 198–208.

LOVAS" R.G., LIOTTAb R.J. , INSOLIA" A., VARGA" K., DELIONd D.S. (1998), MICROSCOPIC THEORY OF CLUSTER RADIOACTIVITY, Physics Reports 294 ;265

Mohammad Shoeb and Sonika (2009), Stability of the s- and p-shell α cluster hypernuclei with strangeness S = −2 to −4 , J. Phys. G: Nucl. Part. Phys. 36; 045104 (15pp).

Routray T.R., Jagajjaya Nayak, Basub D.N.(2009), Cluster radioactivity in very heavy nuclei,:a new perspective , Nuclear Physics A 826; 223–229.

Santhosh K.P., Sabina Sahadevan, Biju R.K.(2009), Alpha radioactivity in heavy and super heavy elements, Nuclear Physics A 825; 159–174 .

Seif W.M., (2006), Decay as a probe of nuclear incompressibility, Phys. Rev. C 74; 034302.

Sham S. Malik", Surender Son K., Satya P. Sud (1997), Preformation factor in cluster radioactivity , Phy. Lett. B 399 ; 8- 12.

Zhang G.L., Lea X.Y. , Zhang H.Q. (2009), Determination of α prefomation for heavy nuclei , Nuclear Physics A 823 ; 16–25.

Sushil Kumar, Shagun Thakur and Rajesh Kumar, (2009), Decay studies of 288−287115 alpha-decay chains,J. Phys. G: Nucl. Part. Phys. 36, 105104 (11pp).

SILI TEANU I., SCHEID W. and SILI TEANU A. O. (2005), ESTIMATES OF ALPHA-DECAY HALF-LIVES OF SUPERHEAVY ELEMENTS, Romanian Reports in Physics, Vol. 57, No. 4, P. 715–746.

Varga K., Lovas R.G. , Liotta R.J., (1992), Absolute alpha-decay widths of 212Po in a combined shell model and cluster model, Phys. Rev. Lett. 69 ; 37

Wildermuth K., Tang Y. C., A Unified Theory of the Nucleus, Academic Press, New York, 1977.