SCHRODINGER EQUATION IN STRING THEORY

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ABSTRACT

String theory is currently considered as the leading candidate for a unified theory of physics combining the Standard Model of forces and particles with gravity which is best described by Einstein theory of General Relativity. Contrary to classical model of point particle, String theory proposes that matter, force, even space and time are composed of tiny vibrating strings. This paper is to elaborate on the correspondence between string states and quantum fields by initially constructing general time-dependent states from string basis states analogous to general time-dependent super-positions of basis states for a point particle. From this derivation we can show that an equation emerges from the 'classical' Schrodinger equation that represents the Schrodinger equation in String theory. This is very interesting to investigate since the Schrodinger equation is at the core of Quantum Mechanics which is the foundation of Standard Model that is the pillar of Nuclear physics.

ABSTRAK

Teori tetali sekarang ini merupakan calon yang paling meyakinkan dalam teori gabungan fizik bagi menyatukan model piawaian daya-daya dan zarah-zarah dengan graviti yang paling tepat di terangkan dengan teori kerelatifan umum Einstein. Berbanding dengan model klasik zarah, teori tetali mencadangkan bahawa jasad, daya malah ruang dan masa itu sendiri pada asasnya terdiri dari tetali-tetali seni yang bergetaran. Kertas kerja ini adalah untuk melanjutkan perbincangan perihal kaitan diantara keadaan tetali dan keadaan medan daya kuantum bermula dengan membina keadaan-keadaan kebergantungan masa yang umum dari keadaan tetali yang asas yang juga boleh di samakan dengan kebergantungan masa umum superposisi keadaan asas bagi zarah. Dari terbitan matematik ini kita boleh menunjukkan bahawa persamaan ini timbul dari persaman Schrodinger 'klasik' di mana ia mewakili persamaan Schrodinger dalam teori tetali. Ini amat menarik untuk di selidiki kerana persamaan Schrodinger adalah pada teras ilmu mekanik kuantum di mana ia merupakan asas kepada model piawaian yakni merupakan tonggak kepada ilmu fizik perihal Nuklear. .

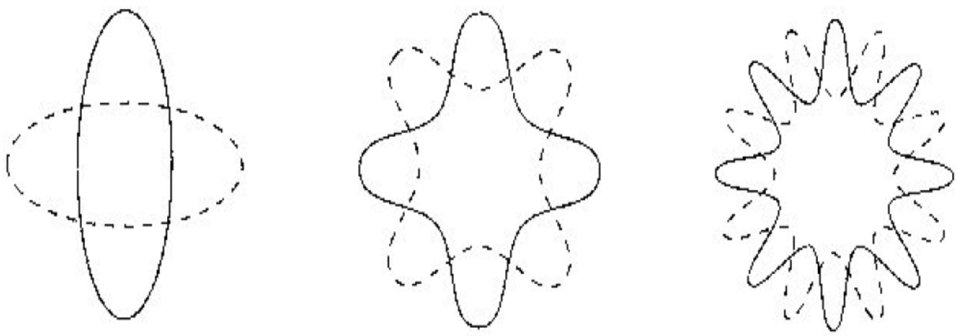
Keywords: Schrodinger, Standard Model, String, foundation of Nuclear Physics

INTRODUCTION

Quantum Mechanics and General Relativity (Wald,1984) are the two most important pillars of modern physics, the former describes physics in the scale of subatomic particles which is the foundation of the standard model and thus the physics of Nuclear, the latter is for the physics of high energy, of astrophysical and cosmological scale. Both have undergone successful test and validation with stunning agreement with experiment. However they are incongruous to each other thus the effort for unification were initiated and the best candidate so far is the String theory.

The origin of String theory in 1960s was very much related in the field of nuclear physics predominantly in the attempt to comprehend the forces that not only bind together proton and neutron inside the nucleus but also binding the quarks that are inside the protons and neutrons which is known as the strong nuclear force (Polchinski,1998) . Rather than point like particles, the theory proposed one-dimensional extended objects called strings which are the most fundamental constituent of matter, force, space and time thus combined them all to a single set of principle or in another word, a unified physics that merge the standard model of elementary particles and general relativity of spacetime.

In relation with nuclear physics, string theory describes the strong interactions where specific oscillation modes of the vibrating strings that can be represented as quantum state corresponds to specific particles (Becker et al, 2007) thus it provides a very unified picture where string as a single fundamental object could explain numerous types of various strong interacting particles which are the building block of nucleus.



Source: (Becker et al, 2007)

Figure 1: Vibrational modes of a string representing numerous types of particles

Classically in nuclear physics the vibrating nature of subatomic particle is well manifest by the most pivotal equation in quantum mechanics that is the Schrodinger equation which was initially derived in the framework of point particle. Since the string theory is also manifest the vibrational nature of particles at the subatomic level it is very interesting to review the nature of Schrodinger equation in the framework of string theory. The derivation can be started in similar method as classical point particle where the coordinate is to be defined, then elaborating on the operator that represent Hamiltonian which having vibrational characteristic namely the Virasoro (Goddard et al, 1986) operator and finally making use the Schrodinger representation in which states are time dependent and satisfy the Schrodinger equation of string.

STRING WAVE EQUATIONS FROM SPACETIME COORDINATE

The coordinate of string in space-time can be represented by

$$X^{\mu}(\tau,\sigma) = x_0^{\mu} + \sqrt{2\alpha'}\alpha_0^{\mu}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{\mu}e^{-in\tau}\cos n\sigma \tag{1}$$

that consist of string tension represented by slope parameter α', initial time like coordinate x_0^{μ}, the 2nd term on the right that represent momentum, time τ and spatial parameter of string σ (where the range is $\sigma \in [0,\pi]$ for open string and $\sigma \in [0,2\pi]$ for close string), zeroth and nth order Fourier mode; α_0^{μ} and α_n^{μ}, thus clearly shows the vibrating nature of string (Zweibach, 2004).

To make the equation more compact and generic we can derive from (1) two linear combinations of derivative, one is differentiated with respect to time, the other is differentiated with respect to space.

We may begin with differentiating (1) with respect to time;

$$\frac{\partial}{\partial\tau}X^{\mu}(\tau,\sigma) = \dot{X}^{\mu}(\tau,\sigma),$$

we have

$$\dot{X}^{\mu}(\tau,\sigma) = \sqrt{2\alpha'}\alpha_0^{\mu} + \sqrt{2\alpha'}\sum_{n\neq 0}\alpha_n^{\mu}e^{-in\tau}\cos n\sigma,$$

more generic of n terms for all integer $\mathbf{Z}$

$$\dot{X}^{\mu}(\tau,\sigma) = \sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\tau}\cos n\sigma \tag{2}$$

then, differentiating (1) with respect to space (spatial parameter of string)

$$\frac{\partial}{\partial\sigma}X^{\mu}(\tau,\sigma)=X^{\mu'}(\tau,\sigma),$$

we have

$$X^{\mu'}(\tau,\sigma)=-i\sqrt{2\alpha'}\sum_{n\neq0}\alpha_n^{\mu}e^{-in\tau}\sin n\sigma$$

and for more generic expression encompassing the whole range of n terms for all integer we have

$$X^{\mu'}(\tau,\sigma)=-i\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\tau}\sin n\sigma\,. \qquad (3)$$

Combining the two linear derivatives of (2) and (3), where for combination by addition

$$\dot{X}^{\mu}(\tau,\sigma)+X^{\mu'}(\tau,\sigma)=\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\tau}\cos n\sigma-i\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\tau}\sin n\sigma$$

and rearrange to extract the exponential term

$$\dot{X}^{\mu}(\tau,\sigma)+X^{\mu'}(\tau,\sigma)=\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\tau}\left(\cos n\sigma-i\sin n\sigma\right),$$

thus

$$\dot{X}^{\mu}(\tau,\sigma)+X^{\mu'}(\tau,\sigma)=\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in(\tau+\sigma)} \qquad (4)$$

and for combination by subtraction

$$\dot{X}^{\mu}\left(\tau,\sigma\right)-X^{\mu'}(\tau,\sigma)=\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\tau}\cos n\sigma+i\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\tau}\sin n\sigma$$

and rearrange to extract the exponential term

$$\dot{X}^{\mu}\left(\tau,\sigma\right)-X^{\mu'}(\tau,\sigma)=\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\tau}\left(\cos n\sigma+i\sin n\sigma\right)$$

.

thus

$$\dot{X}^{\mu}\left(\tau,\sigma\right)-X^{\mu'}(\tau,\sigma)=\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\left(\tau-\sigma\right)}\ . \qquad (5)$$

Combining (4) and (5) we may write

$$\dot{X}^{\mu}\pm X^{\mu'}=\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{\mu}e^{-in\left(\tau\pm\sigma\right)} \qquad (6)$$

This is the solution of the wave equations that satisfy the boundary condition $\dot{X}.X^{'}=0$ where $X^{'}=0$ to satisfy another fundamental condition where open string is attached to brane (Becker *et al*, 2007). In this paper we consider an open string since it represents all the standard model particles except graviton which is the close string.

THE VIRASORO OPERATOR

Now from (1.5), for $\mu=-$

$$\dot{X}^{-}\pm X^{-'}=\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{-}e^{-in\left(\tau\pm\sigma\right)} \qquad (7)$$

for $\mu=I$

$$\dot{X}^{I}\pm X^{I'}=\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^{I}e^{-in\left(\tau\pm\sigma\right)}\ . \qquad (8)$$

So far the light cone coordinates is utilized to represent the motion of strings since the utilization of the light cone coordinate is extensive in string theory calculation. The light cone coordinate is using x^+ and x^- instead of x^0 and x^1. For both open and close string it has been shown (Zweibach, 2004) that

$$n.X(\tau,\sigma) = \beta\alpha'(n.p)\tau \qquad (9)$$

which describes the parameterization of open ($\beta = 2$) and closed ($\beta = 1$) string . By light cone coordinate the relation to X^+ can be shown as

$$n.X = n_\mu X^\mu = \frac{1}{\sqrt{2}}\left(X^0 + X^1\right) = X^+ .$$

Thus, from (9) we may write $n_\mu X^\mu(\tau,\sigma) = \beta\alpha'\left(n_\mu p^\mu\right)\tau$ which then becomes

$$X^+ = \beta\alpha' p^+ \tau \quad . \qquad (10)$$

So it is obvious from (10) that the light cone position of string represented as time dependent only, thus

$$X^{+'} = 0 \quad \text{and} \quad \dot{X}^+ = \beta\alpha' p^+ \quad . \qquad (11)$$

This also implies constraints, moreover by light cone coordinate metric

$$-ds^2 = -2dx^+dx^- + \left(dx^2\right)^2 + \left(dx^3\right)^2$$

we may rewrite in the form of light cone coordinate metric for string (Zweibach, 2004)

$$-2\left(\dot{X}^+ \pm X^{+'}\right)\left(\dot{X}^- \pm X^{-'}\right) + \left(\dot{X}^I \pm X^{I'}\right)^2 = 0$$

or

$$2\left(\dot{X}^{+} \pm X^{+'}\right)\left(\dot{X}^{-} \pm X^{-'}\right) = \left(\dot{X}^{I} \pm X^{I'}\right)^{2} \qquad (12)$$

by (11)

$$2\beta\alpha' p^{+}\left(\dot{X}^{-} \pm X^{-'}\right) = \left(\dot{X}^{I} \pm X^{I'}\right)^{2}$$

thus

$$\dot{X}^{-} \pm X^{-'} = \frac{1}{\beta\alpha' 2p^{+}}\left(\dot{X}^{I} \pm X^{I'}\right)^{2} \quad . \qquad (13)$$

Combining (7), (8) and (13) we have

$$\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_{n}^{-}e^{-in(\tau\pm\sigma)} = \frac{1}{\beta\alpha' 2p^{+}}\left(\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_{n}^{I}e^{-in(\tau\pm\sigma)}\right)^{2}$$

Expand and re- label n terms on the right hand side of the equation to p and q to distinguish the summation terms, thus the equation becomes more generic

$$\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_{n}^{-}e^{-in(\tau\pm\sigma)} = \frac{1}{\beta\alpha' 2p^{+}}\left(\sqrt{2\alpha'}\sum_{p\in\mathbf{Z}}\alpha_{p}^{I}e^{-ip(\tau\pm\sigma)}\right)\left(\sqrt{2\alpha'}\sum_{q\in\mathbf{Z}}\alpha_{q}^{I}e^{-iq(\tau\pm\sigma)}\right) \qquad (14)$$

Considering open string (thus $\beta = 2$) since the open string builds up the standard model of particle that is also the building blocks of nucleus, therefore we may rewrite (14) to

$$\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_{n}^{-}e^{-in(\tau\pm\sigma)} = \frac{1}{2\alpha' 2p^{+}}2\alpha'\sum_{p\in\mathbf{Z}}\alpha_{p}^{I}e^{-ip(\tau\pm\sigma)}\sum_{q\in\mathbf{Z}}\alpha_{q}^{I}e^{-iq(\tau\pm\sigma)} \qquad (15)$$

Combine the summation

$$\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^- e^{-in(\tau\pm\sigma)}=\frac{1}{2p^+}\sum_{p,q\in\mathbf{Z}}\alpha_p^I\alpha_q^I e^{-i(p+q)(\tau\pm\sigma)} \quad . \tag{16}$$

The combined summation can be manipulated and re- label so that the three distinguished terms (n, p, q) can be reduced to just three terms, so by $q\rightarrow n-p$ thus (16) becomes

$$\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^- e^{-in(\tau\pm\sigma)}=\frac{1}{2p^+}\sum_{p,(n-p)\in\mathbf{Z}}\alpha_p^I\alpha_{n-p}^I e^{-in(\tau\pm\sigma)}$$

Similarly we may write

$$\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^- e^{-in(\tau\pm\sigma)}=\frac{1}{2p^+}\sum_{p,n\in\mathbf{Z}}\alpha_p^I\alpha_{n-p}^I e^{-in(\tau\pm\sigma)}$$

so we may extract a definition for the Virasoro term later and capture the p term of summation in bracket

$$\sqrt{2\alpha'}\sum_{n\in\mathbf{Z}}\alpha_n^- e^{-in(\tau\pm\sigma)}=\frac{1}{2p^+}\sum_{n\in\mathbf{Z}}\left(\sum_{p\in\mathbf{Z}}\alpha_p^I\alpha_{n-p}^I\right)e^{-in(\tau\pm\sigma)} \tag{17}$$

then (17) can be reduced to more compact equation

$$\sqrt{2\alpha'}\alpha_n^-=\frac{1}{2p^+}\sum_{p\in\mathbf{Z}}\alpha_{n-p}^I\alpha_p^I \; . \tag{18}$$

This represents a complete solution where we have the expressions for oscillators α_n^- in term of the transverse oscillator of the Fourier mode α_p^I and α_{n-p}^I where summation combination of these transverse oscillators is defined as the transverse Virasoro operator.

$$L_n^\perp=\frac{1}{2}\sum_{p\in\mathbf{Z}}\alpha_{n-p}^I\alpha_p^I \tag{19}$$

THE STRING HAMILTONIAN

The transverse Virasoro operator expression turns out to be very useful in constructing the Schrodinger equation of string since it can be shown later that the Virasoro is physically regards as the total energy or Hamiltonian. From (18) and (19) we rearrange the expressions as a preparation to relate with the Hamiltonian.

$$\sqrt{2\alpha'}\alpha_n^- = \frac{1}{p^+}L_n^\perp \qquad (20)$$

Let see what happen at the most fundamental state of the Virasoro that is at the ground state or at the zero mode, thus (19) becomes

$$L_0^\perp = \frac{1}{2}\sum_{p\in\mathbf{Z}}\alpha_{-p}^I\alpha_p^I \qquad (21)$$

We may expand the equation (21) that covers the whole range of modes

$$L_0^\perp = \frac{1}{2}\alpha_0^I\alpha_0^I + \frac{1}{2}\sum_{p=1}^{\infty}\alpha_{-p}^I\alpha_p^I + \frac{1}{2}\sum_{p=1}^{\infty}\alpha_p^I\alpha_{-p}^I$$

then, the third terms on the right hand side is extended to extract a commutation term

$$L_0^\perp = \frac{1}{2}\alpha_0^I\alpha_0^I + \frac{1}{2}\sum_{p=1}^{\infty}\alpha_{-p}^I\alpha_p^I + \frac{1}{2}\sum_{p=1}^{\infty}\left(\alpha_{-p}^I\alpha_p^I + \alpha_p^I\alpha_{-p}^I - \alpha_{-p}^I\alpha_p^I\right).$$

The commutation between the transverse annihilation oscillator and creation oscillator respectively is derived for the last term

$$L_0^\perp = \frac{1}{2}\alpha_0^I\alpha_0^I + \frac{1}{2}\sum_{p=1}^{\infty}\alpha_{-p}^I\alpha_p^I + \frac{1}{2}\sum_{p=1}^{\infty}\alpha_{-p}^I\alpha_p^I + \frac{1}{2}\sum_{p=1}^{\infty}\left[\alpha_p^I,\alpha_{-p}^I\right],$$

by the commutation relation $\left[\alpha_m^I,\alpha_{-n}^J\right] = m\delta_{m,n}\eta^{IJ}$ (Zweibach, 2004),

$$L_0^{\perp} = \frac{1}{2}\alpha_0^I \alpha_0^I + \sum_{p=1}^{\infty} \alpha_{-p}^I \alpha_p^I + \frac{D-2}{2}\sum_{p=1}^{\infty} p \,.$$

Taking $\eta^{II} = D - 2$ where D represents spacetime (gravitational field) dimension. At the classical level as graviton is spin 2 we may ignore this last term (Zweibach, 2004), thus

$$L_0^{\perp} = \frac{1}{2}\alpha_0^I \alpha_0^I + \sum_{p=1}^{\infty} \alpha_{-p}^I \alpha_p^I$$

It has been defined (Zweibach, 2004) that the string tension represented by slope parameter and string momentum in relation with oscillator of the Fourier zero mode as $\alpha_0^{\mu} = \sqrt{2\alpha'} p^{\mu}$, thus for transverse oscillator of the Fourier zero mode is

$$\alpha_0^I = \sqrt{2\alpha'} p^I$$

Also, it has been defined (Becker et al, 2007) that the relation of oscillator with "classical" oscillator as in Quantum Field Theory (Michio, 1993) which represents annihilation and creation operator respectively; $\alpha_m^{\mu} = a_m^{\mu}\sqrt{m}$, $\alpha_{-m}^{\mu} = a_m^{\mu\dagger}\sqrt{m}$ thus

$$\alpha_p^I = a_p^I \sqrt{p}$$

and

$$\alpha_{-p}^I = a_p^{I\dagger} \sqrt{p} \,.$$

Then, the Virasoro operator $L_0^{\perp}$ can be rewritten in terms of string tension, momentum and the "classical" oscillator as

$$L_0^{\perp} = \alpha' p^I p^I + \sum_{p=1}^{\infty} p a_p^{I\dagger} a_p^I \quad . \qquad (22)$$

The Virasoro operator in term of number operator (Michio,1993) represented as $N^{\perp}$

$$L_0^{\perp} = \alpha' p^I p^I + N^{\perp} \quad , \tag{23}$$

where

$$N^{\perp} = \sum_{p=1}^{\infty} p a_p^{I\dagger} a_p^I \quad . \tag{24}$$

For point particle the Hamiltonian is equivalent to the Virasoro operator where it can be shown that from Heisenberg Hamiltonian in light cone coordinate

$$H = \frac{p^+ p^-}{m^2} \tag{25}$$

but in light cone gauge $X^+ = 2\alpha' p^+ \tau$ and $\frac{\partial}{\partial \tau} X^+ = \frac{p^+}{m^2}$ therefore

$$\frac{p^+}{m^2} = 2\alpha' p^+ \tag{26}$$

thus from (25) and (26) the Hamiltonian expression is

$$H = 2\alpha' p^+ p^- \tag{27}$$

In relation with Virasoro at ground state by let n=0 for (20)

$$\sqrt{2\alpha'}\alpha_0^- p^+ = L_0^{\perp} \tag{28}$$

then since

$$\alpha_0^{\mu} = \sqrt{2\alpha'} p^{\mu}$$

thus

$$\alpha_0^- = \sqrt{2\alpha'} p^- \tag{29}$$

Therefore by (28) and (29)

$$L_0^{\perp} = 2\alpha' p^+ p^- \qquad (30)$$

thus by (27)

$$H = L_0^{\perp} \qquad (31)$$

So the total energy or Hamiltonian is indeed represented by the transverse Virasoro. However the ordering constant $a = 1$ was introduced in String (Michio,1999) due to the existence of extra-dimension as postulated which relates to the Virasoro operator (30) by

$$2\alpha' p^- = \frac{1}{P^+}\left(L_0^{\perp} + a\right)$$

thus

$$L_0^{\perp} - 1 = 2\alpha' p^+ p^-$$

Therefore now (31) becomes

$$H = L_0^{\perp} - 1 \qquad (32)$$

From (23) and (32),

$$H = \alpha' p^I p^I + N^{\perp} - 1 \qquad (33)$$

This is the Hamiltonian that represents a string.

THE SCHRODINGER REPRESENTATION

From point particle framework (Schwarz, 2004) time-dependent state $\left|\psi,\tau\right\rangle$ can be written as

$$\left|\psi,\tau\right\rangle = e^{-iH\tau}\left|\psi\right\rangle \quad . \tag{34}$$

Taking its left side as exponent we may write

$$e^{\ln\left|\psi,\tau\right\rangle} = e^{-iH\tau}\left|\psi\right\rangle$$

then it can be shown

$$\ln\left|\psi,\tau\right\rangle = \frac{H\tau}{i}$$

$$\ln\left|\psi,\tau\right\rangle = \frac{H}{i}\int d\tau = \int\frac{H}{i}d\tau$$

and it can be shown also that

$$\frac{1}{\left|\psi,\tau\right\rangle}\partial\left|\psi,\tau\right\rangle = \frac{H}{i}d\tau\ .$$

Therefore we may write

$$i\frac{\partial}{\partial\tau}\left|\psi,\tau\right\rangle = H\left|\psi,\tau\right\rangle \tag{35}$$

which is the Schrodinger equation of point particle where

$$\left|\psi,\tau\right\rangle = \int dp^{+}d\vec{p}_T\psi(\tau,p^{+},\vec{p}_T)\left|p^{+},\vec{p}_T\right\rangle \tag{36}$$

and the states of quantum point particle is

$$\left| p^+, \vec{p}_T \right\rangle \tag{37}$$

with the point particle wave function

$$\psi(\tau, p^+, \vec{p}_T) \tag{38}$$

Due to isomorphism characteristic of Hilbert spaces (Aharoni et al, 1999), the Schrodinger for the point particle is isomorphic to the classical field equation of a scalar field, therefore the Schrodinger equation of string should also be isomorphic to that of point particle. By (37) the states of string is

$$a_{n_1}^{I_1\dagger} ... a_{n_k}^{I_k\dagger} \left| p^+, \vec{p}_T \right\rangle \;, \tag{39}$$

which consist of multiple Fourier oscillator terms on the left that represent the vibrating strings with the strings wave function

$$\psi_{I_1...I_k}(\tau, p^+, \vec{p}_T)\,. \tag{40}$$

Thus by (39) and (40) which also similar with (36) the time-dependent super-positions of the basis states of string is

$$\left| \psi, \tau \right\rangle = \int dp^+ d\vec{p}_T \psi_{I_1...I_k}(\tau, p^+, \vec{p}_T) a_{n_1}^{I_1\dagger} ... a_{n_k}^{I_k\dagger} \left| p^+, \vec{p}_T \right\rangle \tag{41}$$

then, by (35) and (41) with the Hamiltonian of string (33) we may write

$$i\frac{\partial}{\partial \tau} \int dp^+ d\vec{p}_T \psi_{I_1...I_k} a_{n_1}^{I_1\dagger} ... a_{n_k}^{I_k\dagger} \left| p^+, \vec{p}_T \right\rangle = \left(\alpha' p^I p^I + N^\perp - 1 \right) \int dp^+ d\vec{p}_T \psi_{I_1...I_k} a_{n_1}^{I_1\dagger} ... a_{n_k}^{I_k\dagger} \left| p^+, \vec{p}_T \right\rangle$$

Since the terms of string wave functions on the left side of the equation and Hamiltonian could be coupled with the string wave function on the right side of the equation thus we may rewrite the equation above as

$$\int dp^+ d\vec{p}_T \left(i\frac{\partial}{\partial\tau}\psi_{I_1 \ldots I_k} \right) a_{n_1}^{I_1\dagger} \ldots a_{n_1}^{I_1\dagger} \left| p^+ \vec{p}_T \right\rangle = \int dp^+ d\vec{p}_T \left(\left(\alpha' p^I p^I + N^I - 1 \right) \psi_{I_1 \ldots I_k} \right) a_{n_1}^{I_1\dagger} \ldots a_{n_k}^{I_k\dagger} \left| p^+ \vec{p}_T \right\rangle$$

Extracting from the equation above we have

$$i\frac{\partial}{\partial\tau}\psi_{I_1 \ldots I_k} = \left(\alpha' p^I p^I + N^{\perp} - 1 \right) \psi_{I_1 \ldots I_k}$$

which is the Schrodinger equation that emerges in the string theory framework.

CONCLUSIONS

The Schrodinger equation of string can also be derived in similar manner as that of the classical point particle due to isomorphism between string wave functions and quantum field wave functions. In this paper we have started the derivation by introducing the most fundamental definition in a physical analysis which is the coordinate in space and time that represents a string. The coordinate consist not only the space and time but also their relation with string characteristics such as the string tension and vibrational characteristic. The coordinate equation had been derived further representing more generic form, satisfying the constraints of open strings and obtains the Virasoro operator that not only exhibit the oscillatory behavior of strings but also representing the string Hamiltonian or total energy. We have chosen the open string in this calculation since the open string builds up the standard model of particle that is also the building blocks of nucleus. Then, by expressing the string wave function and consequently the time-dependent super-positions of the basis states of string the Schrodinger equation in the string theory framework had been derived.

ACKNOWLEDGEMENT

I would like to thank University Malaya on the short-term research grant awarded for the research, and to my employer, the management of PROTON R&D with gratitude for the encouragement and allow me to venture into research field far beyond the automotive world.

REFERENCES

Ahorany, O., Gubser, S. S., Maldacena, J., Ouguri, H. and Yaron, O. (1999). *Large N-Fields Theories, String Theory and Gravity*.hep-th/9905111v3.

Becker, K., Becker, M. and Schwarz, J.H. (2007). *String Theory and M-Theory a Modern Introduction.* Cambridge: Cambridge University Press.

Goddard, P., and Olive, D.I. (1986). Kac-Moody and Virasoro Algebras in Relation to Quantum Physics. *Int. J. Mod. Phys.* A1, 303.

Michio, K. (1999). Introduction to Superstring and M-theory. 2nd ed. New York: Springer-Verlag.

Michio, K (1993). Quantum Field Theory a Modern Introduction. New York: Oxford University Press.

Polchinski, J. (1998), *String Theory.*(in two volume). Cambridge: Cambridge University Press.

Schwarz, F. (2004) *Advance Quantum Mechanics*. New Delhi: Springer-Verlag.

Wald, R. M (1984), *General Relativity.* Chicago: University of Chicago Press.

Zweibach, B. (2004). *A First Course in String Theory*. Cambridge: Cambridge University Press.