STUDY THE DYNAMIC APERTURE OF A COMPACT HADRON DRIVER FOR CANCER THERAPY

Leo Kwee Wah[1], Ken Takayama[2], Toshikazu Adachi[2], Tadamichi Kawakubo[2] and Tanuja Dixit[3]

[1]*Nuclear Agency, Bangi 43000 Kajang. Selangor. Malaysia.*
[2]*High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki, Japan.*
[3]*Society for Applied Microwave Electronics Engineering & Research (SAMEER), Mumbai, India.*
e-mail: leo77@nm.gov.my

ABSTRACT

A design of a compact hadron driver for future cancer therapies based on the induction synchrotron concept is given. In order to realize a slow extraction technique in a fast-cycling synchrotron, which allows the energy sweep beam scanning, the zero momentum-dispersion D(s) region and high flat D(s) region are necessary. The present design meets both requirements. The lattice has the two-fold symmetry with a circumference of 52.8 m, 2 m-long dispersion-free straight section, and 3 m-long large flat dispersion straight section. Assuming a 1.5 T bending magnet, the ring can deliver heavy ions of 200 MeV/au at 10 Hz. Details of the lattice parameters and dynamic aperture approach and method are studied are discussed.

ABSTRACT

Reka bentuk pemacu hadron padat untuk terapi kanser masa hadapan berdasarkan konsep induksi synchrotron diberikan. Untuk merealisasikan teknik pengekstrakan perlahan dalam segerak berbasikal pantas, yang membolehkan pengimbasan rasuk sapuan tenaga, rantau D(s) penyebaran momentum sifar dan kawasan D(s) rata tinggi adalah perlu. Reka bentuk sekarang memenuhi kedua-dua keperluan. Kekisi mempunyai simetri dua lipatan dengan lilitan 52.8 m, keratan lurus bebas serakan sepanjang 2 m, dan keratan lurus serakan rata besar 3 m panjang. Dengan mengandaikan magnet lentur 1.5 T, gelang boleh menghantar ion berat 200 MeV/au pada 10 Hz. Butiran parameter kekisi dan pendekatan dan kaedah apertur dinamik dikaji dibincangkan.

Keywords: cancer therapies, synchrotron, energy sweep beam scanning

INTRODUCTION

A lattice design of a compact hadron driver for future cancer therapies is given as shown in figure 1 has been designed [1]. Whereby the lattice has the two-fold symmetry with a circumference of 52.8 m, 2 m long dispersion free straight section, and 3 m long large flat dispersion straight section [1][2]. The lattice parameter is a design based on the linear term of motion equation. Hence, the individual common components will be treated as 2 X 2 matrices as the following equations [3][4].

For drift space,

$$Matrix\ Drift = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \tag{1}$$

For focusing magnet,

$$\textit{Matrix Focusing}\begin{bmatrix} \cos\left[\sqrt{k}L\right] & 1/\sqrt{k}\sin\left[\sqrt{k}L\right] \\ -\sqrt{k}\sin\left[\sqrt{k}L\right] & \cos\left[\sqrt{k}L\right] \end{bmatrix} \tag{2}$$

For defocusing magnet,

$$\textit{Matrix Defocusing}\begin{bmatrix} \cosh\left[\sqrt{|k|}L\right] & 1/\sqrt{|k|}\sinh\left[\sqrt{|k|}L\right] \\ \sqrt{|k|}\sinh\left[\sqrt{|k|}L\right] & \cosh\left[\sqrt{|k|}L\right] \end{bmatrix} \tag{3}$$

For bending magnet,

$$\textit{Matrix Bending} = \begin{bmatrix} \cos[\theta] & \rho\sin[\theta] \\ -\sin[\theta] & \cos[\theta] \end{bmatrix} \tag{4}$$

where:

$k = k - \textit{value of the related magnets}$

$L = \textit{Length of the related magnets}$

For perturbation term, which can be treated as nonlinear term as an essential term must be evaluated. So that the desired beam can beam accelerated inside the accelerator ring with the desired lattice. Therefore, the dynamic aperture as one of the essential parameters of the particles must be considered and will be discussed in the following section.

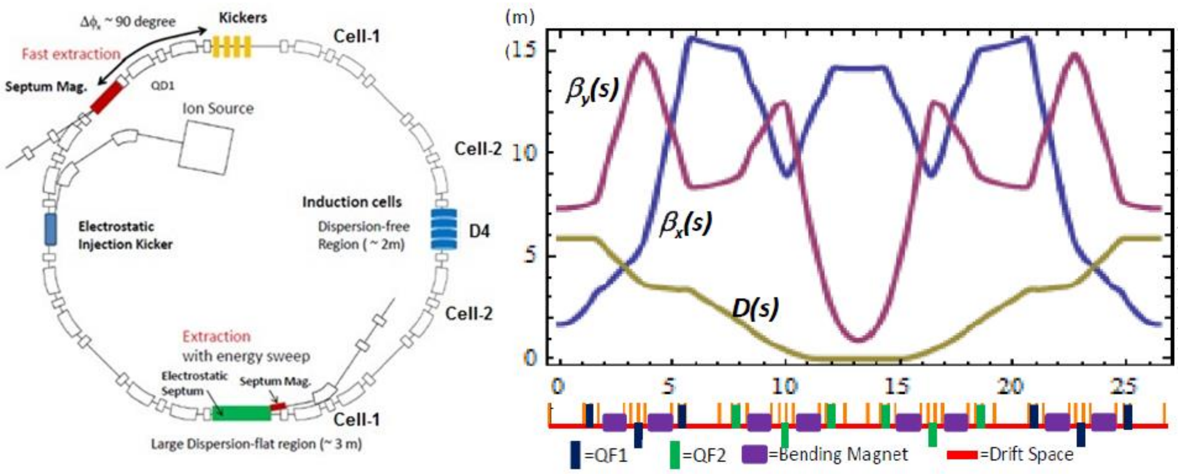

Figure 1. Outline of the ring (left) with half ring beta function (right) [1]

DYNAMIC APERTURE

In general, the motion of particle of a synchrotron is dominated by the betatron oscillation equation as shown in left-hand side of Eqs [5][6]. Without any perturbations, the beam trajectory is ideal with the movement at the orbit according to the lattice designed. One of the parameters of the lattice designed is the dynamic aperture, which measures the motion stability of charged particles in a global manner and is able to evaluate the physical size of the ring. During the long-term stability, tiny dynamical diffusion can lead the initial stable orbit slowly into an unstable region especially the charged particles must be ensuring the stability over billions of turns. Therefore, the basic method for evaluating the dynamic aperture through a computing tracking code. A model of the ring (lattice) must be built within the code that includes an integration routine for each magnetic element. Then the particle trajectory is tracked with its stability evaluation [7].

For horizontal direction,

$$\frac{d^2x}{ds^2} + K_x(s)X = F(s') \tag{5}$$

For vertical direction,

$$\frac{d^2y}{ds^2} + K_y(s)Y = F(s') \tag{6}$$

To evaluate the dynamic aperture of the compact hadron driver, the simulation approach is discussed at next section.

SIMULATION TRACKING MODEL

Basically, the ring is referred to as the lattice which comprises of element magnets such as bending magnets, quadrupole magnets. The arrangement of the magnets is the so-called lattice of the ring. For the simulation tracking model as shown in figure 2, the non-linear perturbation terms will be introduced within the six segments at the bending magnet as indicated by the red arrow, meanwhile a non-linear perturbation at the exist of the quadrupole magnets as indicated by the blue arrow will be introduced as well. Therefore, the total perturbation as shown Table 1 will be considered as a kick which is a collective effect that will interrupt the trajectory of the particles from the initial turn to over billions of turns.

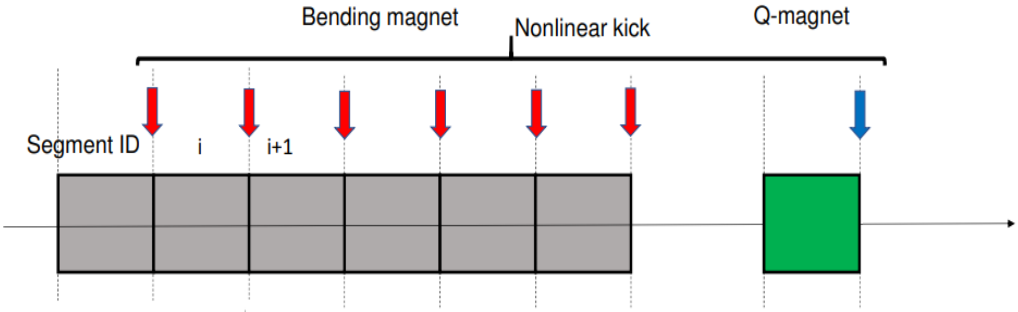

Figure 2. Perturbation terms of the magnets

Table 1. Number of the perturbation as the kick to the particles

No.	Type of Magnets	Number of Magnets	Number of kicks
1	Bending magnet	16	96
2	Quadrupole magnet	20	20

Typically, the particles orbit in phase space can be treated by the transfer matrixes (M_{i+1}) as shown in Eqs. [1][2][3][4] can be derived as,

Horizontal direction,

$$\begin{pmatrix} x \\ p_x \end{pmatrix}_i^{exit} = M_{i+1} \begin{pmatrix} x \\ p_x \end{pmatrix}_i^{entrance} \tag{7}$$

Vertical direction,

$$\begin{pmatrix} y \\ p_y \end{pmatrix}_i^{exit} = M_{i+1} \begin{pmatrix} y \\ p_y \end{pmatrix}_i^{entrance} \tag{8}$$

By introducing the perturbation term as a kick, the transfer matrixes can be derived as:

Horizontal direction,

$$\begin{pmatrix} x \\ p_x \end{pmatrix}_i^{entrance} = \begin{pmatrix} x \\ p_x \end{pmatrix}_i^{exit} + \begin{pmatrix} 0 \\ \Delta p_x\left(x_i^{exit}, y_i^{exit}\right) \end{pmatrix} \tag{9}$$

Vertical direction,

$$\begin{pmatrix} y \\ p_y \end{pmatrix}_i^{entrance} = \begin{pmatrix} y \\ p_y \end{pmatrix}_i^{exit} + \begin{pmatrix} 0 \\ \Delta p_y\left(x_i^{exit}, y_i^{exit}\right) \end{pmatrix} \tag{10}$$

Whereby, the kick can be derived as,

Horizontal direction,

$$\Delta p_x\left(x_i^{exit}, y_i^{exit}\right) = -\frac{\lambda_{mag}\Delta B_x\left(x_i^{exit}, y_i^{exit}\right)}{B\rho} \tag{11}$$

Vertical direction,

$$\Delta p_y\left(x_i^{exit}, y_i^{exit}\right) = -\frac{\lambda_{mag}\Delta B_y\left(x_i^{exit}, y_i^{exit}\right)}{B\rho} \tag{12}$$

where,

segment length can de expressed as,

$$\lambda_{mag} = \begin{cases} \lambda_B = t_B/6 \\ \lambda_B = t_Q \end{cases} \tag{13}$$

The perturbation terms are contributed from the magnet's simulation [8] can be expressed as:

For bending magnet:

Horizontal direction,

$$\Delta p_x\left(x_i^{exit}, y_i^{exit}\right) = -\frac{LB_y(0,0)\left\{a\left(x_i^2 - y_i^2\right) + b\left(x_i^4 - 6x_i^2y_i^2 + y_i^4\right)\right\}}{6B\rho} \tag{14}$$

Vertical direction,

$$\Delta p_y\left(x_i^{exit}, y_i^{exit}\right) = -\frac{LB_y(0,0)\left\{2ax_iy_i + 4bx_iy_i\left(x_i^2 - y_i^2\right)\right\}}{B\rho} \tag{15}$$

For Quadrupole magnet:

Horizontal direction,

$$\Delta p_x\left(x_i^{exit}, y_i^{exit}\right) = -Lk\left\{ax_i^5 - 10x_i^3y_i^2 + 5ax_iy_i^4 + by_i^9 - 36bx_iy_i^2 + 126bx_i^5y_i^4 - 84bx_i^3y_i^6 + 9bx_iy_i^8\right\} \tag{16}$$

Vertical direction,

$$\Delta p_x\left(x_i^{exit}, y_i^{exit}\right) = -Lk\left\{5ax_i y_i^4 - 10x_i^2 y_i^3 + ay_i^5 + 9bx_i^8 y_i - 84bx_i^6 y_i^3 + 126bx_i^4 y_i^5 - 36bx_i^2 y_i^7 + by_i^9\right\} \quad (17)$$

where [8]:

$p_{x,y}$ = momentum of the particle at horizontal or vertical direction

a and b = Constant coefficient of the polynomial function determined by the least square fitting method for the non-linear term of the magnet simulation [8]

By development of the algorithm with using Eqs [1] until [17] and constant coefficient of a and b ban be fitted as shown in Table 3, the particles tracking simulation have been obtained and evaluated as described in next section.

SIMULATION TRACKING RESULT

The simulation procedure as shown in figure 3, firstly the initial distribution and parameters of the particles are defined, then the individual transfer matrices of the magnets and perturbation terms (considered as kick) will be prepared. Finally, the well-defined particles will be transported through the entire ring by using the prepared transfer matrices with the dedicated turns and the selected position along the ring. Finally, the trajectory of the particles at the phase space will be plotted horizontally and vertically.

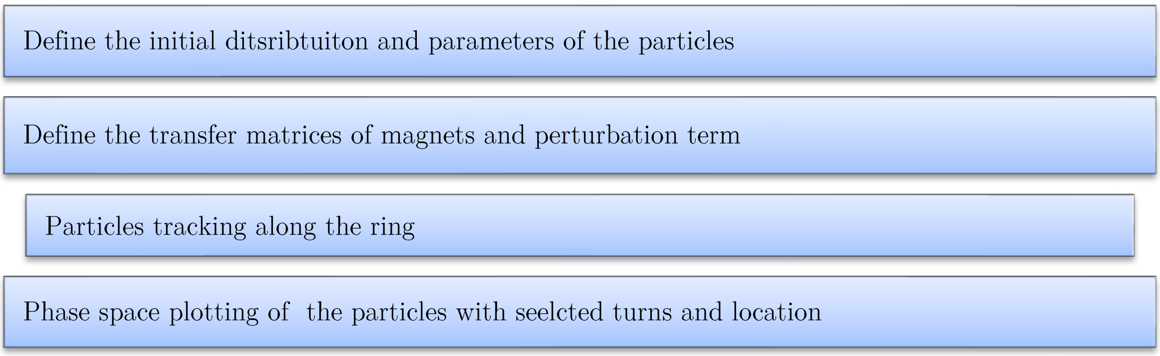

Figure 3. Simulation procedure

For this study, three cases with injection energy of 1200 keV, initial distribution and parameters are defined as shown in Table 2.

Table 2: 3 cases with the initial distribution and perturbation constant coefficient

Case	Horizontal initial distribution	Vertical initial distribution	a_b [m^{-2}]	b_b [m^{-4}]	a_Q [m^{-4}]	b_Q [m^{-8}]
1	0.04, 0	0.01 -0.04	6.2010 X 10^{-2}	5.5620 X 10^{1}	4.4068 X 10^{2}	-3.6857 X 10^{6}
2	0.04, 0	0.01 -0.04	6.2010 X 10^{-2}/4	5.5620 X 10^{1}/4	4.4068 X 10^{2}/4	-3.6857 X 10^{6}/4
3	0.04, 0	0.01 -0.04	6.2010 X 10^{-2}/8	5.5620 X 10^{1}/8	4.4068 X 10^{2}/8	-3.6857 X 10^{6}/8

Three cases will be evaluated horizontally and vertically as shown in phase space plot Figure 4, 5, 6, 7, 8 and 9.

Case 1

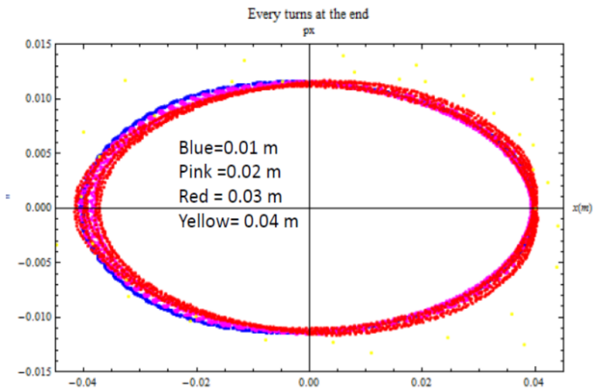

Figure 3. Horizontal phase space of Case 1 at end of the ring

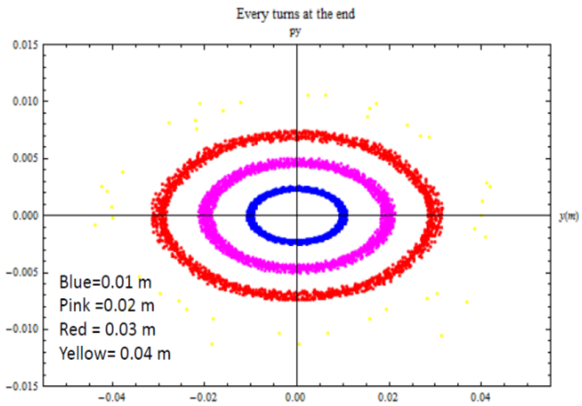

Figure 4. Vertical phase space of Case 1 at end of the ring

Case 2

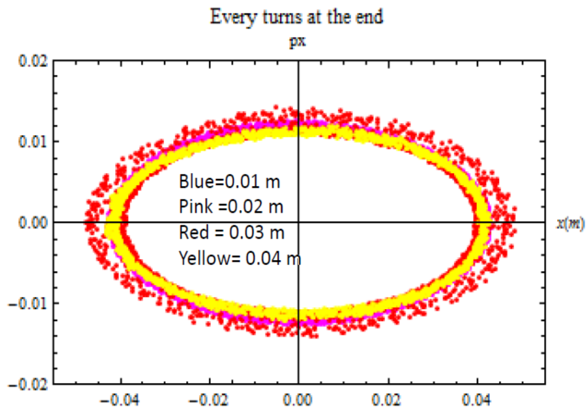

Figure 5. Horizontal phase space of Case 2 at end of the ring

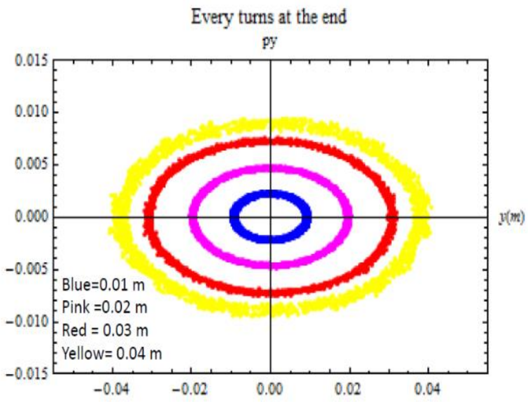

Figure 6. Vertical phase space of Case 2 at end of the ring

Case 3

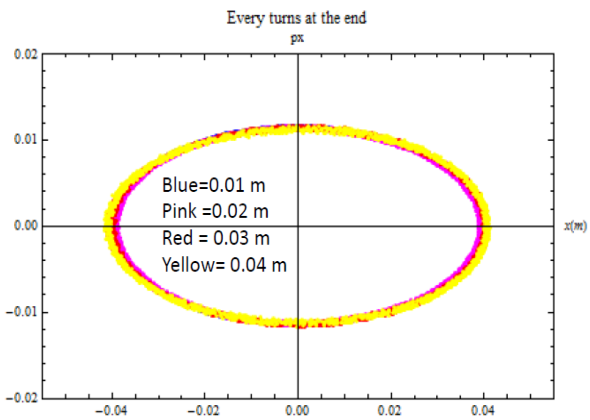

Figure 7. Horizontal phase space of Case 3 at end of the ring

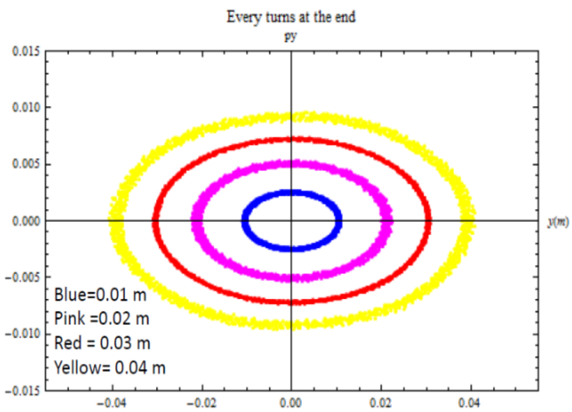

Figure 8. Vertical phase space of Case 3 at end of the ring

DISCUSSION

In general, the approach based on the simulation model which can evaluate the dynamic aperture as referred to the stable region of the particles within the beam.

Table 3: 3 cases with the achievable stable region

Case	a_b [m^{-2}]	b_b [m^{-4}]	a_Q [m^{-4}]	b_Q [m^{-8}]	Stable Region X & Y [m]
1	6.2010 X 10^{-2}	5.5620 X 10^{1}	4.4068 X 10^{2}	-3.6857 X 10^{6}	~ 0.01
2	6.2010 X 10^{-2}/4	5.5620 X 10^{1}/4	4.4068 X 10^{2}/4	-3.6857 X 10^{6}/4	~ 0.02
3	6.2010 X 10^{-2}/8	5.5620 X 10^{1}/8	4.4068 X 10^{2}/8	-3.6857 X 10^{6}/8	~ 0.04

According to Figure 3, 4, 5, 6, 7 and 8, we noticed that two dynamic apertures are changed with three different cases as summarized in Table 3. The perturbation term will contribute significant effect to the dynamic aperture, which must be minimized to ensure the beam stability within the physical size of the ring beamline. The original perturbation constant coefficient as evaluated in Case 1, which only has the stable region within 1 cm. With the reduction of perturbation constant coefficient with 4 times as shown in Case 2, the stable region in increased to 2 cm. Meanwhile, the stable region able to achieve 4 cm with 8 times reduction of the original perturbation constant coefficient. Therefore, the non-linear term of the magnets [8] must be reduced. For such manner, the magnets must be redesigned with the repeating process of the dynamic aperture survey until it attains the desired results.

CONCLUSION

In this paper, the approach of dynamic aperture is introduced with its evaluation. The magnets design is essential to achieve a much larger stable region and ensure the energetic beam able to delivery to the user facility. In the near future, the magnets need to be redesigned and dynamic aperture of the ring must be evaluated again.

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