THE STRUCTURE AND LOW-LYING ENERGETIC STATES OF $^{160\text{-}162}$Dy ISOTOPES.

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ABSTRACT

$^{160-162}Dy$ nuclei are the best candidates to study collective properties of low-lying states since these nuclei are quite well studied experimentally. Phenomenological model is used to evaluate the positive-parity states energy spectra and the structure of these nuclei by taking into account the Coriolis mixing between states. Deviations from the adiabatic conditions are found to be occurred.

ABSTRAK

$^{160-162}Dy$ teras ialah calon terbaik untuk belajar ciri-ciri kolektif negeri-negerirendah sejak teras ini agak dikaji dengan baik secara eksperimen.Model fenomenologi ialah digunakan untuk menilai keadaan parity-positif yang menyatakan spektrum tenaga dan struktur teras ini yang mengambil kira keadaan-keadaan Coriolis yang bercampuran. Penyimpangan dari syarat-syarat adiabatik ialah didapati telah berlaku.

Keywords: Phenomenological model, nuclear particle,

INTRODUCTION

The low-lying, collectively magnetic dipole excitations in deformed nuclei are discovered in the last decade [1]. Since then, interest to study the properties of the deformed nuclei has increased especially in the last few years [2]. It is evidently to state that the low-lying 1^+ states spread around the excitation energy of 3 MeV in energy spectrum [3]. Taking account into the Coriolis mixing of the isovector collective M1 states with low-lying states will lead for the non-adiabaticity of electromagnetic properties to occur [4-5].

$^{160-162}Dy$ nuclei are the best candidates to study collective properties of low-lying states [6]. With much new experimental work being carried out, the comparison from the previous researches and further work can be done so well. They are quite well studied experimentally [7-9].

The Model

The basic states of the Hamiltonian include ground (gr), β_n $(0^+_{\beta_1}, 0^+_{\beta_2})-$, $\gamma-$vibrational and $K^\pi = 1^+_\nu$ rotational bands. As n is the number of included $\beta-$vibrational states, so ν is the number of 1^+ collective states.

-The nuclear Hamiltonian is written in the two-partition form

$$H = H_{rot}(I^2) + H^{\sigma}_{K,K'}(I) \tag{1}$$

$H_{rot}(I^2)$ is the rotational part and

$$H^{\sigma}_{K,K'}(I) = -\omega_K \delta_{K,K'} - \omega_{rot}(I)(j_x)_{K,K'}\chi(I,K)\delta_{K,K'\pm 1} \tag{2}$$

$(j_x)_{K,K'}$ is the matrix element describing the Coriolis coupling of rotational bands and $\omega_{rot}(I)$ is the angular frequency of core rotation yielded from

$$\omega_{rot}(I) = \frac{dE_{cor}(I)}{dI}$$

ω_K is the head energy of respective K^+ bands which is the lowest energy level with $I = 0$ and

$$\chi(I,0) = 1, \quad \chi(I,1) = \left[1 - \frac{2}{I(I+1)}\right]^{\frac{1}{2}}$$

wave function of the nuclear Hamiltonian

$$\psi^I_\nu = \sum_K \phi^I_{\nu,K} \left| IMK \right\rangle \tag{3}$$

where $\phi^I_{\nu,K}$ represents the Coriolis mixing coefficient of basis states and

$$\left| IMK \right\rangle = \sqrt{\frac{2I+1}{16\pi^2}} \left\{ \sqrt{2}\psi^I_{gr,K} D^I_{M,0} + \sum \frac{\psi^I_{K',K}}{\sqrt{1+\delta_{K',0}}} \left[D^I_{M,K'}(\theta) b^+_{K'} + (-1)^{I+K'} D^I_{M,-K'}(\theta) b^+_{-K'} \right] \right\} \tag{4}$$

$\psi^I_{K',K}$ are the amplitudes of basis states mixing from the $(4+\nu)$ bands includes the ground $\left|0\right\rangle$ states bands and the single-phonon $b^+_{\lambda=2}\left|0\right\rangle = b^+_K \left|0\right\rangle$ with all the mentioned rotational bands before.

By solving the Schrodinger equation

$$H^{\sigma}_{K,n}\psi^I_{K',n} = \varepsilon^{\sigma}_n \psi^I_{K,n} \tag{5}$$

We obtained wave function and energy of states with positive parity.

Total energy of states is determined by following

$$E^{\sigma}_n(I) = E_{rot}(I) + \varepsilon^{\sigma}_n(I) \tag{6}$$

Energy of rotational core $E_{rot}(I)$ can be determined by different methods. In this research, we used Harris parameterization of the angular momentum and energy [10].

$$E_{rot}(I) = \frac{1}{2}\Im_0 \omega^2_{rot}(I) + \frac{3}{4}\Im_1 \omega^4_{rot}(I) \tag{7}$$

$$\sqrt{I(I+1)} = \Im_0 \omega_{rot}(I) + \Im_1 \omega^3_{rot}(I) \tag{8}$$

where $\Im_0$ and $\Im_1$ are the adjustable inertial parameters of rotational core. A method of defining the even-even deformed nuclei inertial parameters using the experimental data up to $I \le 8\hbar$ for ground band is suggested in recent paper [11] and quoted in Table 1. The linear dependencies of moment of inertia for states $J_{eff}(I)$ on the square of angular frequency of rotation $\omega_{eff}(I)$ are plotted in Figure 1.

The rotational frequency of the core, $\omega_{rot}(I)$ is calculated by solving the cubic equation, which the real root is as follows:

$$\omega_{rot}(I)=\left\{\frac{\tilde{I}}{2\Im_1}+\left(\left(\frac{\tilde{I}}{2\Im_1}\right)^2+\left(\frac{\Im_0}{3\Im_1}\right)^3\right)^{\frac{1}{2}}\right\}^{\frac{1}{3}}+\left\{\frac{\tilde{I}}{2\Im_1}-\left(\left(\frac{\tilde{I}}{2\Im_1}\right)^2+\left(\frac{\Im_0}{3\Im_1}\right)^3\right)^{\frac{1}{2}}\right\}^{\frac{1}{3}}. \qquad (9)$$

where $\tilde{I}=\sqrt{I(I+1)}$. Equation (9) gives value of $\omega_{rot}(I)$ at the given spin *I*.

Explaining the Coriolis interaction, given the value of the perturbed (experimental) energies, $E^1_{\exp}$ and $E^2_{\exp}$ it is possible to calculate the interaction matrix element $\omega_{rot} j_x$ from the pure energies E^1_{theor} and E^2_{theor}, such that

$$\begin{pmatrix} E^1_{theor} & \omega_{rot} j_x \\ \omega_{rot} j_x & E^2_{theor} \end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}=E^{1,2}_{\exp}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}. \qquad (10)$$

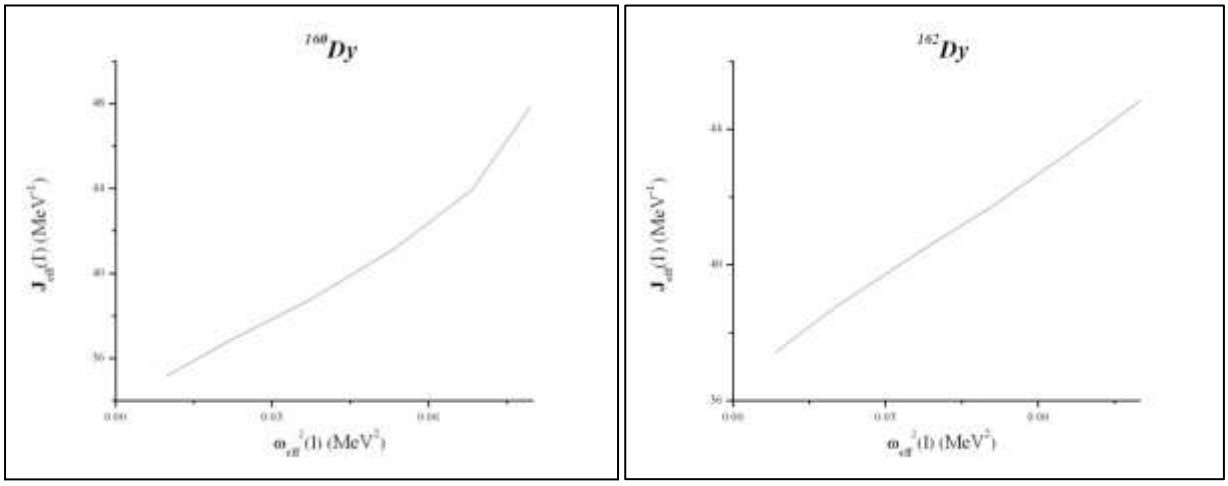

Figure 1. The linear dependencies of $J_{eff}(I)$ on $\omega^2_{eff}(I)$.

Table 1. Inertial parameters of rotational core used in the calculations.

Nucleus	$\Im_0(MeV^{-1})$	$\Im_1(MeV^{-3})$
^{160}Dy	33.96	131.07
^{162}Dy	36.61	105.77

RESULTS

$\Im_{eff}(I)$ is plotted as a function of $\omega^2_{eff}(I)$ at low spin, $I \le 8\hbar$. The linear dependency of $\Im_{eff}(I)$ on $\omega^2_{eff}(I)$ is invalidating at higher spin. Figure 1 illustrates the linear dependency of $\Im_{eff}(I)$ on $\omega^2_{eff}(I)$ is at low spin, $I \le 8\hbar$ for isotopes $^{160-162}Dy$.

The values of the inertial parameters, $\Im_0$ and $\Im_1$ obtained are tabulated in Table 1 for isotopes $^{160-162}Dy$. The parameter $\Im_0$ placed in the second column of the tables represents the moment of inertia of the ground

states band [12]. Moment of inertia is a parameter used to measure the resistance of an object to its motion changes.

The parameters fitted with the model are presented in Table 2. The lowest energy for ground-state and β_n bands were taken from experimental energies, since they are not affected by the Coriolis forces at spin $I = 0$:

$$\omega_{gr} = E_{gr}^{expt}(0) \text{ and } \omega_{\beta_n} = E_{\beta_n}^{expt}(0).$$

The headband energies for the collective 1^+ states in $^{160-162}Dy$ nuclei are taken from experimental values since the $K^\pi = 1^+$ bands for these nuclei have been observed experimentally. Coriolis rotational states mixing matrix elements $(j_x)_{K,K'}$ and $\gamma-$band head energies ω_γ were determined by using the least square fitting method of the diagonalizable matrix.

$$\begin{pmatrix} \omega_K - \varepsilon & \omega_{rot} j_x \\ \omega_{rot} j_x & \omega_{1^+} - \varepsilon \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \omega_K \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

Table 2. Parameters used in the calculations.

Nucleus	ω_{β_1}	ω_{β_2}	ω_1	ω_γ	$(j_x)_{gr,1}$	$(j_x)_{\beta_1,1}$	$(j_x)_{\beta_2,1}$	$(j_x)_{\gamma,1}$
^{160}Dy	1.280	1.444	1.775	0.879	1.689	2.170	2.224	2.031
^{162}Dy	1.400	1.666	1.720	0.807	0.077	0.103	0.108	0.093

Table 3 and 4 give the calculated Coriolis mixing coefficients, $\phi_{\nu,K}^I$ which represents the mixture components of other bands in certain band. Structure of $^{160-162}Dy$ can be understood by these calculated values.

In ^{160}Dy isotope , the mixing component of 1^+ bands in gr band at spin $I = 2\hbar$ is the lowest followed by the mixing components of 1^+ bands in $\gamma-$ band and β_2- band .The mixing component of 1^+ band in β_1- band is the highest one. Same situations occur as the spin increased. But, the mixing component of 1^+ band in $\gamma-$ band at spin $I = 12\hbar$ is higher than that in gr band.

For ^{162}Dy, the mixing component of 1^+ band in gr band at spin $I = 2\hbar$ is the lowest followed by the mixing component of 1^+ band in $\gamma-$ and β_1- bands. The mixing component of 1^+ band in β_2- band is strikingly higher than others. The mixing component of 1^+ band in β_1- band is also considerably higher compared to that in $\gamma-$ and gr bands. As expected, as the spin increased to $I = 12\hbar$, the mixing components of 1^+ band in the low-lying band states increased.

Table 3. Structure of ^{160}Dy states.

I	gr	0_{β_1}	0_{β_2}	1^+	γ	gr	0_{β_1}	0_{β_2}	1^+	γ
	Ground-state band					β_1				
2	0.9975	0.0082	0.0075	0.0688	0.0091	-0.0288	0.9240	0.2187	0.2962	-0.0989
4	-0.9906	-0.0263	-0.0240	-0.1275	-0.0337	-0.0659	0.8016	0.3511	0.3635	-0.3127
6	0.9773	0.0523	0.0477	0.1878	0.0683	-0.0857	0.7143	0.3737	0.3281	-0.4849
8	-0.9557	-0.0836	-0.0765	-0.2487	-0.1090	0.0931	-0.6579	-0.3680	-0.2807	0.5867
10	-0.9259	-0.1171	-0.1076	-0.3073	-0.1513	0.0959	-0.6223	-0.3593	-0.2433	0.6444
12	-0.8898	-0.1497	-0.1381	-0.3608	-0.1913	0.0969	-0.5991	-0.3520	-0.2158	0.6791
	γ					β_2				
2	-0.0194	0.0514	0.0378	0.1397	0.9879	0.0205	0.3016	-0.9207	-0.2410	0.0540
3	-	-	-	0.1921	0.9814	-	-	-	-	-
4	-0.0771	0.1667	0.1272	0.2947	0.9291	0.0284	0.4921	-0.8450	-0.1873	0.0891
5	-	-	-	0.2846	0.9586	-	-	-	-	-
6	-0.1588	0.2619	0.2077	0.3946	0.8410	0.0304	0.5602	-0.8091	-0.1444	0.0988
7	-	-	-	0.3472	0.9378	-	-	-	-	-
8	-0.2491	0.3136	0.2564	0.4392	0.7622	0.0311	0.5894	-0.7919	-0.1187	0.1025
9	-	-	-	0.3913	0.9203	-	-	-	-	-
10	-0.3388	0.3362	0.2808	0.4490	0.7012	0.0314	0.6045	-0.7825	-0.1023	0.1042
11	-	-	-	0.4235	0.9059	-	-	-	-	-
12	-0.4216	0.3430	0.2907	0.4398	0.6532	0.0316	0.6135	-0.7768	-0.0909	0.1052

We cannot analyze the mixing components of $\beta_n -$ and $\gamma -$ bands in ^{160}Dy isotope in terms of the headband energy ω_K easily because the headband energies of the band look random. But, after careful analysis, the headband energy difference between $\beta_n -$bands is smaller compared to the difference between the headband energy of $\gamma -$and $\beta_2 -$bands. This explains why the mixing components of $\beta_1 -$ band in $\beta_2 -$ band and vice versa are higher and increasing as the spin increased.

As we see before in ^{162}Dy isotope, the mixing component of 1^+ band in $\beta_2 -$ band is strikingly higher than others and the mixing component of 1^+ band in $\beta_1 -$ band is also considerably higher compared to that in $\gamma -$ and gr bands. This can be explained by investigating the effects of the closeness of the headband energy, ω_K between two bands on the structure of nuclei. The headband energy of $\beta_2 -$and $\beta_1 -$bands are the closest headband energy to the headband energy of 1^+ bands.

The theoretical energy spectra of positive-parity states in $^{160-162}Dy$ are presented in Figure 2 and 3 respectively in comparison with the experimental energies experimentally [7-9]. From Figure 2, we see that energy difference $\varepsilon(I) = \left|E_{theor}(I) - E_{\exp}(I)\right|$ of the β_n- bands of ^{162}Dy isotope increase with the increase in the angular momentum I. At high spin, I the nonadiabaticity of energy rotational bands occurs. Two states with same spin, I and parity, π from different bands cross in that region causes Coriolis mixing. We predict the existence of s-band states to perturb the pure β_1- band states due to large deviations from the experimental energies.

Table 4. Structure of ^{162}Dy states.

	gr	0_{β_1}	0_{β_2}	1^+	γ	gr	0_{β_1}	0_{β_2}	1^+	γ
I	Ground-state band					β_1				
2	1.0000	0.0000	0.0000	0.0030	0.0000	0.0001	-0.9998	-0.0006	-0.0213	0.0002
4	-1.0000	0.0000	0.0000	-0.0053	-0.0001	0.0002	-0.9993	-0.0018	-0.0379	0.0007
6	-1.0000	-0.0001	-0.0001	-0.0074	-0.0001	0.0005	-0.9986	-0.0035	-0.0529	0.0013
8	-1.0000	-0.0001	-0.0001	-0.0093	-0.0002	0.0008	-0.9978	-0.0055	-0.0665	0.0021
10	0.9999	0.0002	0.0002	0.0110	0.0003	0.0011	-0.9969	-0.0077	-0.0786	0.0030
12	0.9999	0.0003	0.0002	0.0125	0.0004	0.0014	-0.9959	-0.0100	-0.0895	0.0039
	γ					β_2				
2	0.0000	0.0001	0.0000	0.0055	1.0000	0.0004	0.0033	-0.9917	-0.1285	0.0008
3	-	-	-	0.0086	1.0000	-	-	-	-	-
4	-0.0001	0.0002	0.0002	0.0114	0.9999	0.0012	0.0099	-0.9764	-0.2156	0.0026
5	-	-	-	0.0139	0.9999	-	-	-	-	-
6	-0.0003	0.0005	0.0003	0.0163	0.9999	0.0021	0.0183	-0.9595	-0.2810	0.0049
7	-	-	-	0.0186	0.9998	-	-	-	-	-
8	-0.0004	0.0007	0.0005	0.0208	0.9998	0.0032	0.0271	-0.9440	-0.3287	0.0073
9	-	-	-	0.0228	0.9997	-	-	-	-	-
10	-0.0006	0.0011	0.0008	0.0247	0.9997	0.0041	0.0359	-0.9308	-0.3637	0.0097
11	-	-	-	0.0265	0.9996	-	-	-	-	-
12	-0.0008	0.0014	0.0010	0.0283	0.9996	-0.0051	-0.0443	0.9198	0.3898	-0.0119

Other than this mentioned obvious deviation, the experimental positive-parity states energy spectra are reproduced. But at higher spin, I the theoretical energies deviate from the observed energies suggests the nonadiabaticity of energy rotational bands. Few new states and collective 1^+ band are predicted. Insufficient number of states of rotational bands especially β_2- band make it difficult to see the occurrence of nonadiabaticity of energy rotational β_n- bands at high spin. However, few states still can be predicted by the model.

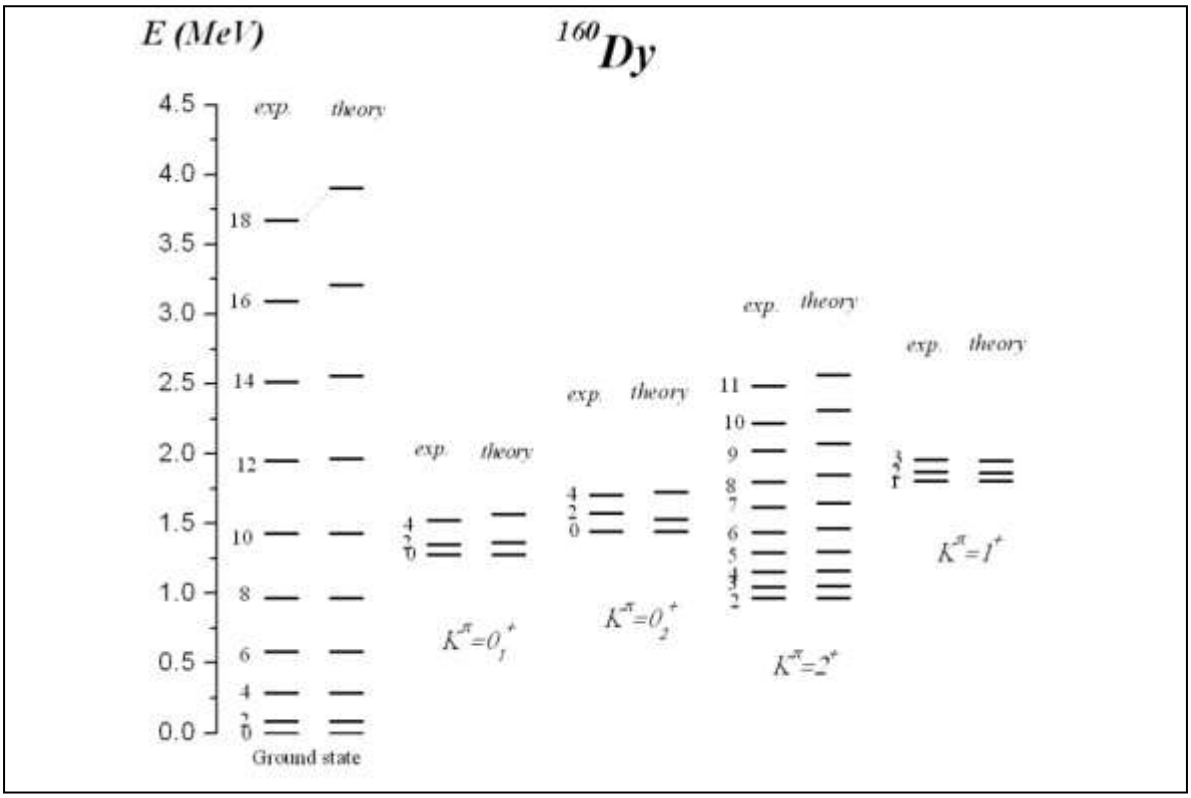

Figure 2. Energy spectrum of positive-parity states of ^{160}Dy isotope.

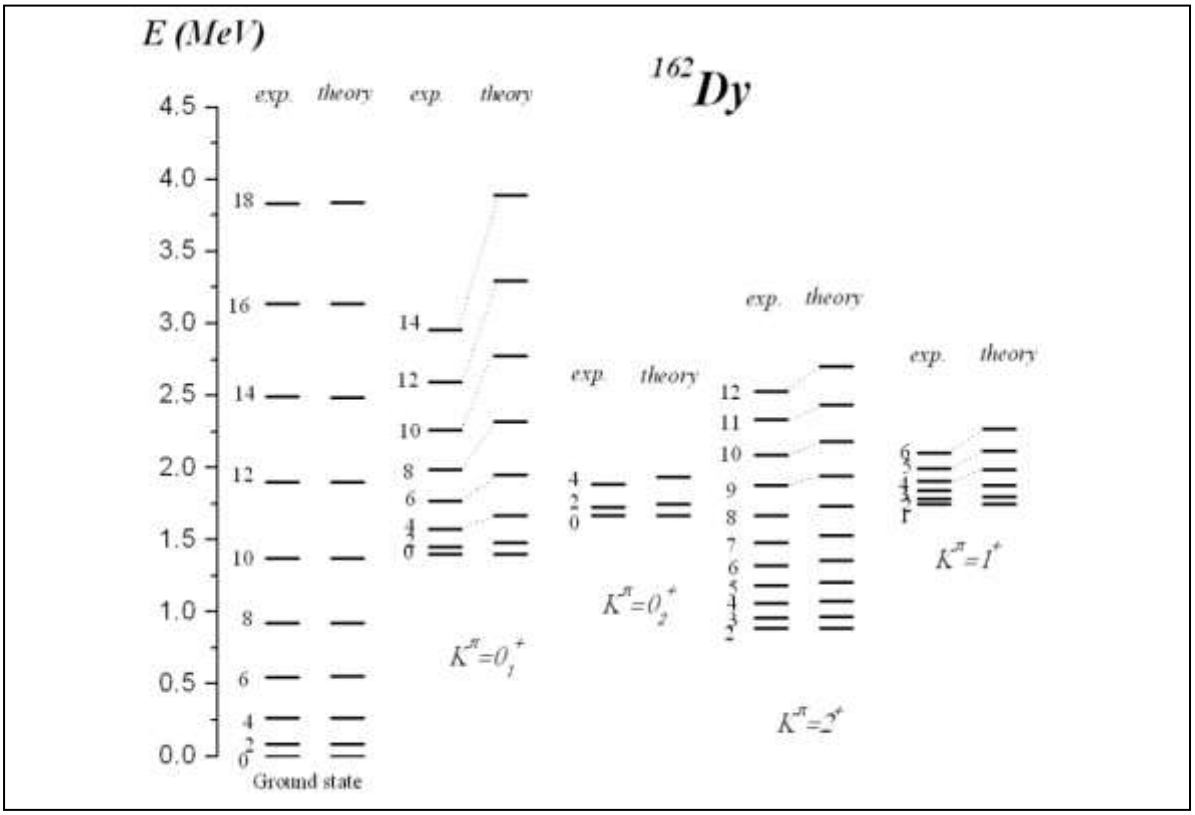

Figure 3. Energy spectrum of positive-parity states of ^{162}Dy isotope.

CONCLUSIONS

This work is based on the phenomenological model [4-5], which shows the deviation of the energy spectrum of positive parity states in even-even deformed nuclei from the adiabatic theory. Energy spectra for the isotopes $^{160-162}Dy$ were calculated and the results showed are in good agreement with the experimental data. At high spin I, the law of $E_{rot}(I) \sim I(I+1)$ [6] is violated. The calculations are done by taking into account the Coriolis mixing of positive parity states. The mixing components of the states is represented by the calculated values of Coriolis mixing coefficients, $\phi^{I}_{\nu,K}$. The value of the mixing component explained why deviation occurred. The strength of states mixing is influenced by the values of Coriolis interaction matrix elements, $(j_x)_{K,K'}$ and headband energy, ω_K. With the agreement between the theoretical and experimental data, few states that never been observed experimentally are predicted.

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